Vertically Propagating Rossby Waves

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1. Introduction

Vertically propagating waves have an important role in the dynamics of the stratosphere. The waves interact with the stratosphere's mean circulation, maintain the energy and momentum budget of the stratosphere, transport trace species (e.g. ozone) concentrations between the troposphere and stratosphere, and are the reason for the phenomena known as sudden stratospheric warming. The waves originate in the lower troposphere where they are forced to propagate vertically by topographic forcing or land-sea diabatic heating differences. There are two distinct regions of vertically propagating waves, the extratropics and the tropics. Quasi-stationary Rossby waves are the most significant vertically propagating mode in the extratropics, while Kelvin and mixed Rossby-gravity waves are the most prominent vertically propagating waves in the tropics (Holton, 1980). Through mechanical modeling and observations it has been found that these vertically propagating waves occur only under certain atmospheric conditions. Namely, they are most prominent in the northern hemisphere winter under weak westerly wind conditions (Charney and Drazin, 1961). Only the longest waves, i.e. wave number 1 and 2, are able to propagate beyond the troposphere into the stratosphere, as will be show quantitatively later. Figure 1 provides a remarkable example of the influence the extratropical waves have during the northern hemisphere wintertime. In the wintertime the cyclonic vortex in the 30 mb geopotential height is greatly distorted by a ridge of high pressure, the Aleutian high, which is composed of wavenumbers 1 and 2 (compare bottom wintertime picture with top summertime picture). Labitzke and van Loon (1972) found similar waves to the wave number 1 of the northern hemisphere in the southern hemisphere. The lack of prominent vertically propagating extratropical waves in the southern hemisphere is most likely due to smaller difference in land-sea heating contrasts and difference in topographic forcing.

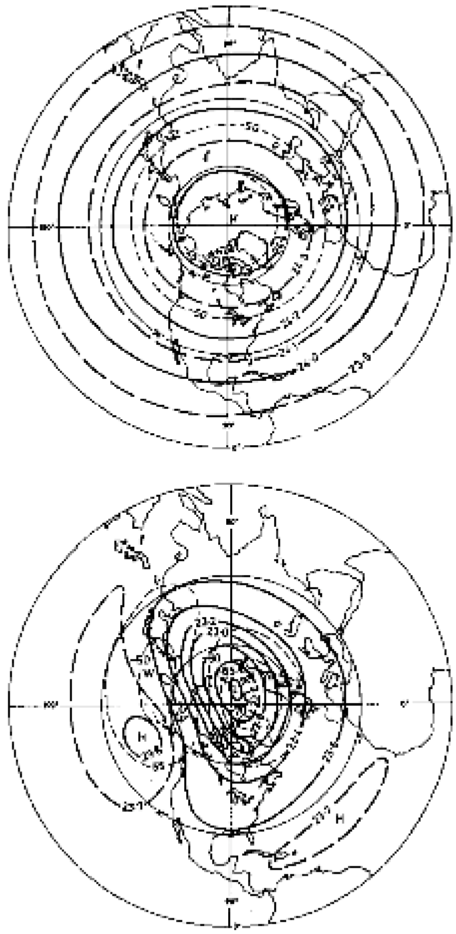


Figure 1. Seasonal mean 30 mb geopotential height (solid lines, km) and temperatures (dashed lines, °C) fields for summer (top) and winter (bottom) Northern Hemisphere seasons (After Hare, 1968)

This paper will focus on the dynamics of the extratropical vertically propagating Rossby waves. The next section will discuss quantitatively under what conditions stationary Rossby waves are able to propagate into the stratosphere. Section 3 will discuss the structure of Rossby waves in the vertical with an emphasis on how the waves propagate through the use of waveguides. Section 4 reviews the role critical levels/lines have on Rossby wave propagation. Section 5 briefly mentions the observed phenomenon of sudden stratospheric warming due to the presence of Rossby waves. Finally, section 6 concludes the paper.

2. Criteria for Vertical Propagation of Rossby Waves

Following Holton (2004) chapter 12.3, the criteria under which Rossby waves will propagate vertically is examined. Charney and Drazin (1961) first explained the confinement of vertically propagating waves to the winter hemisphere. While their analysis used spherical coordinates, Holton chooses to use a mid-latitude β-plane, where $\beta = \frac{df}{dy} = constant$, to take advantage of Cartesian coordinates while still retaining the essential physics for the analysis. To begin, the quasi-geostrophic potential vorticity (QGPV) equation is written,

$$\frac{dq}{dt} = 0 \quad (1)$$

with

$$q \equiv f + \nabla^2\psi + \frac{f^2}{N^2}\rho_o^{-1}\frac{\partial}{\partial z}\left(\rho_o \frac{\partial \psi}{\partial z}\right).$$

The careful reader will notice this is the same as Pedlosky QGPV equation 6.5.21, except the vertical coordinate is defined in log-pressure coordinates as $z \equiv -H\ln\left(\frac{p}{p_s}\right)$, where H is a constant scale height. The waves are manifested as a small-amplitude disturbance superposed on a constant zonal-mean flow. The flow is therefore broken up into a mean state and a perturbation state, such that $\psi = \overline{\psi} + \psi'$ and $q = \overline{q} + q'$. If the decompositions are subbed into equation (1) and linearized the perturbation q' field must satisfy

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)q'+\beta\frac{\partial\psi'}{\partial x}=0 \quad (3)$$

where

$$q'\equiv\nabla^2\psi'+\frac{f_o^2}{N^2\rho_o}\frac{\partial}{\partial z}\left(\rho_o\frac{\partial\psi'}{\partial z}\right).$$

Equation (3) has separable solutions of the form,

$$\psi'(x,y,z,t)=\Psi(z)e^{i(kx+ly-kc_x t)+z/2H} \quad (4)$$

where c_x is the zonal phase speed. Subbing (4) into (3) gives

$$\frac{d^2\Psi}{dz^2}+m^2\Psi=0 \quad (5)$$

where

$$m^2\equiv\frac{N^2}{f_o^2}\left[\frac{\beta}{(\bar{u}-c_x)}-\left(k^2+l^2\right)\right]-\frac{1}{4H^2}$$

Assuming the Rossby waves are stationary, such that c_x= 0 and remembering that m^2 must be positive for waves to propagate vertically, we see that vertically propagating modes can only exist when the mean zonal flow satisfies the condition

$$0<\bar{u}<\beta\left[\left(k^2+l^2\right)+f_o^2/\left(4N^2H^2\right)\right]^{-1}\equiv U_c \quad (6)$$

where U_c is the Rossby critical velocity. The critical velocity says that vertical propagation of Rossby waves can only occur in the presence of westerly winds weaker than a critical value that depends on the horizontal scale of the waves. Note that it is safe to assume stationary Rossby waves, as their primary forcing is stationary, i.e. topography, and no critical knowledge is lost through the assumption. Equation 6 also verifies that only the longest waves are able to propagate into the stratosphere. Consider the variables in the second term on the right hand side of the inequality to be constant, for the entire term to be large, that is larger than $\overline{u}$, k and l must be small, meaning long wavelengths are necessary. Alternatively, consider the phase speed of Rossby waves as a means to understand why vertical propagation is restricted to wave numbers 1 and 2. Figure 2 is a dispersion diagram for mid-latitude Rossby waves. Line a represents the phase speed for large wavenumber, small wavelength waves, while line b is the phase speed for a wave of the same frequency but a smaller wavenumber and hence longer wavelength. The key difference in the two is the slope of their lines. Line b is much steeper than a, making the phase speed greater. The 'fast' wave speed allows the wave to propagate against the mean wind. When the mean field is easterly the Rossby waves are not 'strong' enough to propagate into the stratosphere. The dispersion diagram also reveals the group velocity as the slope of lines tangent to the 'circles.' On the diagram the small arrows show the group velocity propagation.

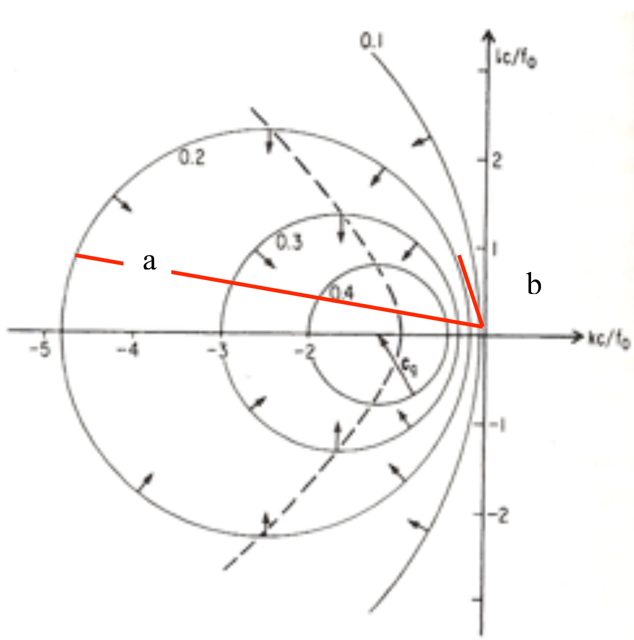


Figure 2 – Dispersion diagram for Rossby waves. Omega is the y-axis while k is the x-axis. (Gill1982)

3. Rossby Wave Structure in the Vertical

The amplitude of Rossby waves increases and shifts phase to the west with height. This is readily seen by examining the streamfunction,

$$\psi \propto e^{imz} e^{\frac{z}{2H}}. \quad (7)$$

We see that ψ does not vary with height simply as e^{imz} but instead increases with height with the additional factor of $e^{\frac{z}{2H}}$. The reason for the increase in amplitude is simply the reduction of atmospheric density as height increases. Figure 3 below illustrate the vertical structure of wavenumbers 1 and 2 for January. For the interested reader, similar diagrams, along with the mathematical derivation of ψ may be found in Holton (1975) pg. 120-125.

Careful examination of figure 3 also reveals the confinement of the vertical wave structure near the axis of the westerly winter hemisphere jet (c.f. 3 & 4). In figure 4 we see that in the lower stratosphere, ~ 10 mb, the jet axis becomes deflected from ~ 45°N to ~ 60°N due to the intrusion of the summer hemisphere easterlies into the winter hemisphere. This deflection of the jet is known as the ‘polar night jet’ and becomes the ‘wave guide’ for vertically propagating Rossby waves. The ‘waveguide’ should be thought of as a channel, which the waves are directed through and allowed to propagate higher than the troposphere.

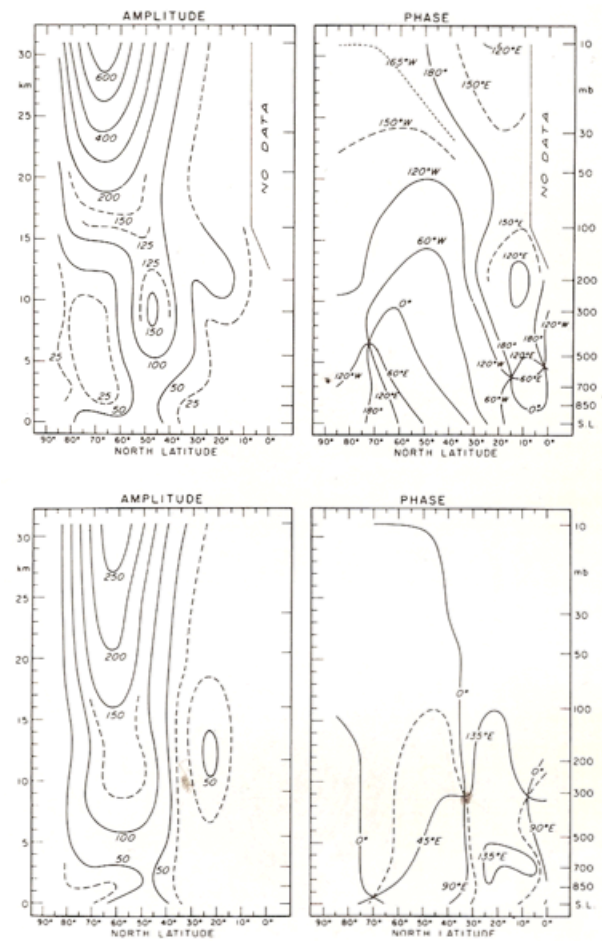


Figure 3 – Latitude-height sections of zonal standing waves of wavenumbers 1 (top) and 2 (bottom) in January. Amplitude is in meters. Phase is longitude of ridge (after van Loon et al., 1973).

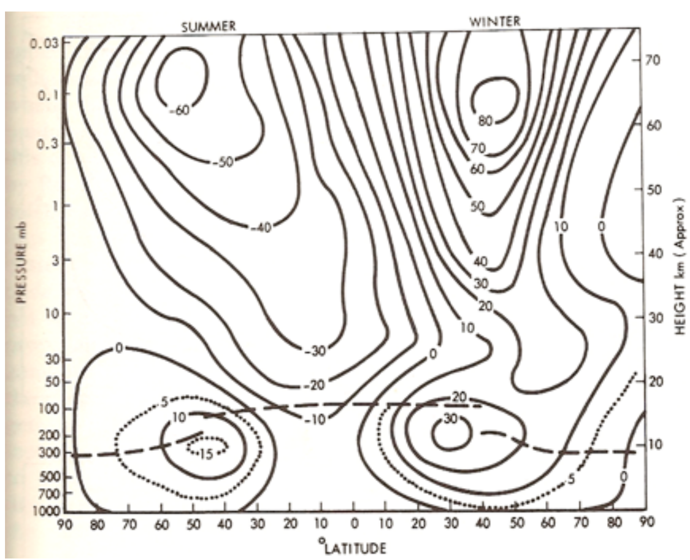


Figure 4 – Schematic latitude-height sections of the mean zonal wind (m/s) at the solstices (From Holton 1975 pg. 5. After Murgatroyd, 1969a)

As Holton (1975) points out, the 'guiding' of the Rossby waves into the region of strong mean zonal winds is one of the most important features to the dynamics of the winter stratosphere. For this reason, we will look in detail at this phenomenon by following Karoly and Hoskins (1981). The common method by which Rossby waveguides is understood is through ray theory. It is assumed that the waves act similar to a light ray and are refracted in a manner similar to the familiar Snell's law. The waves become trapped in the region of strong mean zonal winds and are guided upward. To understand this visually, picture how a radar waveguide works. EM radiation pulses are generated by the transmitter, travel through rectangular shaped metal channels by continually being refracted and reflected off the channel walls, and become emitted to space when the antenna is reached (see fig. 2.1 Rinehart 2004). To understand this mathematically, Karoly and Hoskins begin with a set of quasi-geostrophic equations of motion on a sphere using pressure as the vertical coordinate. After linearizing about a basic state, the equations are reduced to a single equation of the streamfunction (see their eqn. 1). They then derive a wave equation by applying a Mercator or cylindrical projection of the sphere and non-dimensional variables to their equation 1. A Mercator projection is a transfer of coordinates from spherical to Cartesian coordinates. This is frequently done when representing the world on flat maps. Imagine the spherical points being projected onto a cylinder and then rolled flat. Their wave equation for a baroclinic atmosphere is then,

$$\left(\frac{\partial}{\partial t} + \bar{u}_M \frac{\partial}{\partial x}\right)\left(\nabla_M^2 \Psi + \alpha^2 \frac{\partial^2 \Psi}{\partial z^2} - \alpha^2 n^2 \Psi\right) + \beta_M \frac{\partial \Psi}{\partial x} = 0, \quad (8)$$

where $\bar{u}_M$ is the mean basic zonal flow, ∇_M^2 is the Laplacian, and $\alpha^2 = f_M^2 a^2 / \left(N^2 H_o^2\right)$ with a as the radius of the Earth, N^2 the Brunt-*Väisälä* frequency, and H_o the scale height. Ψ is the scaled streamfunction, while $n^2 = \frac{1}{4} - H_o N \frac{\partial N^{-1}}{\partial z} + H_o^2 N \frac{\partial^2 N^{-1}}{\partial z^2}$ acts like an index of refraction due to its dependence on the variation of the Brunt-*Väisälä* frequency with height, i.e. how the stratification or density is changing with height. Finally, β_M represents the meridional gradient of QGPV on the sphere. Assuming that $\bar{u}_M$ and β_M are *slowly* varying, the dispersion equation for plane wave solutions

$$\Psi = Ae^{i(kx+ly+mz-\omega t)} \quad (9)$$

of (8) is

$$\omega = \bar{u}_M k - \frac{\beta_M k}{k^2 + l^2 + \alpha^2\left(m^2 + n^2\right)}. \quad (10)$$

The group velocity for each component, k, l, m, is then derived, so that a ray may be defined in the direction of the local group velocity. This is allowed because the 'activity (?)' of the wave moves with the group velocity, which means the wave activity propagates along a ray with speed equal to the group velocity. Then the variation of the wave frequency and wavenumber along a ray is solved. Noting that the dispersion relation has no explicit dependence on x and t, therefore making k and ω constant along a ray, the meridional and vertical wavenumber variation may be written as,

$$\frac{d_g l}{dt} = -\frac{\partial \omega}{\partial y} \qquad \frac{d_g m}{dt} = -\frac{\partial \omega}{\partial z}. \quad (11)$$

In the above equation $\frac{d_g}{dt}$ is the full derivative moving with the group velocity. By integrating the individual group velocities as well as equation (11), wave rays for a given frequency and basic state may then be solved. Karoly and Hoskins do not go through the integration. The key point is that knowing the dispersion relationship the rates of change of l and m along a ray are known, therefore making it possible to represent the wave as a single ray. Knowing the wave may be represented as a ray, they seek to examine the behavior of the ray (wave) as it propagates. The position of the ray is given by an angle between the horizontal and the ray, i.e.

$$\tan\theta = \frac{w_g}{v_g} = \frac{m}{l}. \quad (12)$$

If a particular frequency is considered and the total wavenumber for that given frequency, $K_\omega = \left(k^2 + l^2 + \alpha^2 m^2\right)^{\frac{1}{2}}$, is defined, the rate of change of θ following a ray is given by,

$$\frac{d_g\theta}{dt} = \frac{K_\omega}{K_\omega^2 - k^2}\hat{i} \bullet \vec{c}_g' \times \nabla K_\omega, \quad (13)$$

where $\hat{i}$ is a unit vector in the x-direction and $\vec{c}_g'$ is the group velocity in the y-z plane. Note that in K_ω^2, k is a constant as we are only considering propagation in the y-z plane. Because the group velocity is crossed with the gradient of the total wavenumber, equation (13) tells us that rays are always refracted towards the direction of ∇K_ω. The schematic below from Hoskins and Ambrizzi (1993) shows the aforementioned result very nicely. Figure (5a) shows how the rays are refracted toward higher values of K_s, while (5b) shows how waves get trapped when a local maximum in K_s is present, thus providing a Rossby waveguide because the waves is always trying to ‘dive’ towards higher values of K_s. In the Hoskins Ambrizzi paper they note how local maximums in K_s are likely to occur in strong westerly jets. Due to the curvature of the flow β_M tends to have a larger relative maximum than $\overline{u}_M$ in equation (10), which explains why the presence of the polar night jet in the winter time tropopause is able to act as a Rossby waveguide.

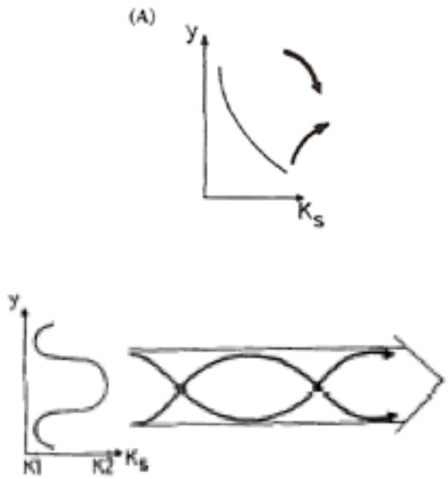
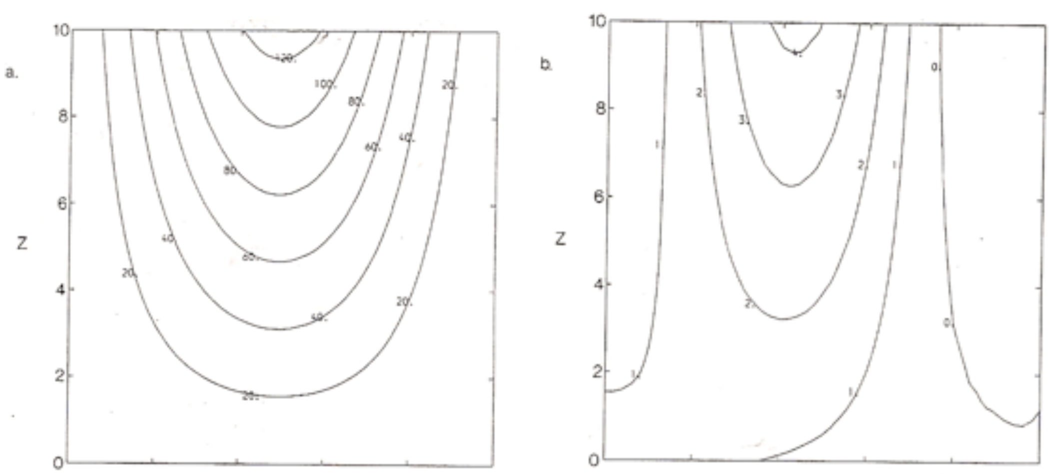


Figure 5 – Schematic stationary Rossby wavenumber (K_s) profiles and ray path refraction. K_s is shown as a function of y and schematic ray paths are shown by heavy lines with arrowheads. (A) simple refraction and (B) waveguide effect of a K_s maximum. Hoskins and Ambrizzi (1993)

As a note, the Hoskins and Ambrizzi (1993) paper considered horizontally propagating Rossby waves, but the idea is exactly the same and it is strongly recommended that the reader consider their paper when trying to understand vertical propagation, as their explanation is very clear and the mathematical derivations are cleaner. Karoly and Hoskins apply the ray theory derived in equations (8) through (13) to a simple basic state to simulate how the waves propagate and get trapped. A simple model of the stratosphere is used with the zonal wind varying between 40° and the pole with a maximum at 65° and a uniform vertical shear. Angular velocity, temperature, and static stability are all assumed to be constant. The zonal flow is shown in figure 6a. Knowing the zonal wind, the vorticity gradient, β_M , can be solved and is plotted in 6b. Notice how the maximum in the vorticity gradient is sifted slightly equatorward to 62°.

The shift arises due to the simple fact that the vorticity gradient is largest near the equator and decreases toward the poles. The stationary wavenumber, i.e. solving for K_s by setting ω = 0 in equation (10), is then plotted in 6c. There is a local maximum in K_s near 54° above z = 8, on the equator side of the jet maximum, but elsewhere there is a general decrease in K_s as latitude and height increase. The additional shift to the equator in K_s maximum is due to the parameters α and n wrapped up in the definition of K_s. This shift is important because it reveals how the waves will propagate on the 'edges' of the jet and not through the jet. Recall how weak westerly winds are required for vertical propagation, so that the waves cannot penetrate the jet max. Rather, the jet max acts as the 'channel wall,' which the waves will be guided along, as will be shown in figure 7. On all three graphs, the height is given as the scale height. As a reference ~ 2.5 is the location of the tropopause.



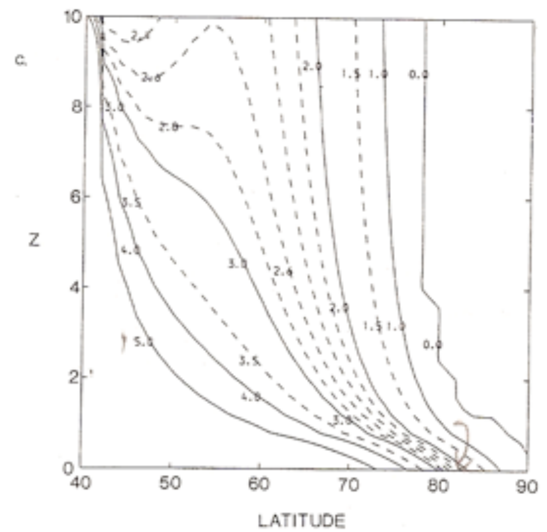


Figure 6 – The basic state for the simple model of the stratosphere, (a) zonal wind, (b) vorticity gradient, β_M *, and (c) stationary wavenumber,* K_s*. (Karoly and Hoskins 1982)*

Under the aforementioned basic state, the wave rays are then solved for (see Karoly and Hoskins 1982 section 3 for details). The rays for zonal wavenumber 1 from a source at 55°, z = 0.77 are shown in Figure 7. Comparison of figure 7 with figure 6c reveals how the rays are refracted toward higher values of K_s, consistent with what equation (13) predicts. A few features to point out are 1) the predominance of most of the wave rays curving at the tropopause, 2) the curious refraction of the last wave ray on the far right and 3) the wave ray that propagates up to z = 10. The first is caused simply by most of the rays being refracted towards the equator before being caught up in the local maximum in the stratosphere. The second occurs due to the nature of ∇K_s near the surface, as seen in figure 6c. ∇K_s has a tight gradient at the surface as the pole is approached, causing rays that tend toward that region to be refracted downwards. Lastly and perhaps most importantly, one of the wave rays is able to propagate high into the atmosphere because the ray gets caught in the region of local K_s maximum and becomes trapped (recall how the wave ray got caught in the K_s

maximum in fig. 5b). This confirms the theory that strong westerly jets can act as waveguides. Karoly and Hoskins go on to examine the wave ray nature in more realistic basic states, which the reader is advised to consider.

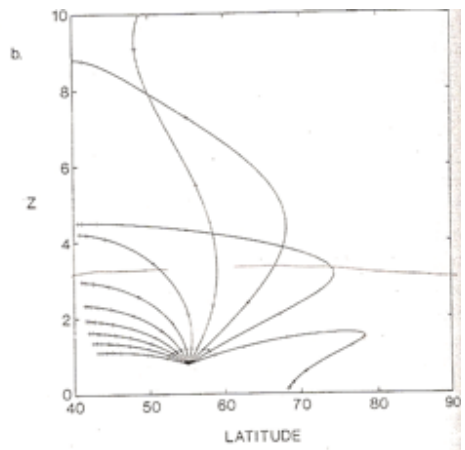


Figure 7 – Rays for zonal wavenumber 1 in the simple stratospheric basic state. The source is at 55°. (Karoly and Hoskins 1982)

At this point it should be clear under what conditions and why and how Rossby waves are able to propagate high into the atmosphere. Next, we will consider what happens to Rossby waves as they reach a point where they are no longer able to propagate upward, such a level is known as the critical level/line.

4. Critical Level for Rossby Waves

As the critical level is approached the linear analysis breaks down and non-linear effects must be considered to explain how the vertical wavelength approaches zero as the wave approaches the critical level. Holton (2004) chapter 12.3.2 provides the clearest discussion of the nature of Rossby waves at the critical level, i.e. wavebreaking. Wavebreaking is the rapid, irreversible deformation of the waves. Holton considers isolines of potential vorticity on isentropic surfaces as material contours. Wavebreaking is then examined by considering how the material contours behave in the potential vorticity field. Because non-linear effects should be considered Holton begins by breaking the potential vorticity field expressed in equation (1) into a mean and disturbance state,

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)q'+v'\frac{\partial\overline{q}}{\partial y}=-u'\frac{\partial q'}{\partial x}+v'\frac{\partial q'}{\partial y}, \text{ (14)}$$

where the right hand side represents the non-linear terms. As always, it is much simpler to deal with only the linear terms. Therefore, a simple scale analysis will be used to prove that it is safe to neglect the non-linear terms. If steady waves that propagate relative to the ground with zonal wave speed, c_x ,are assumed, it can be shown that for the variation of phase in time and space, $\phi = k(x - c_x t)$,

$$\frac{\partial}{\partial t}=c_x\frac{\partial}{\partial x}. \text{ (15)}$$

Equation (15) tells us that in the linearized version of equation (14) there is a balance between advection of the disturbance potential vorticity q' by the Doppler-shifted mean wind and the advection of mean potential vorticity by the disturbance meridional wind, i.e.

$$\left(\overline{u}-c_x\right)\frac{\partial q'}{\partial x}=-v'\frac{\partial\overline{q}}{\partial y}. \text{ (16)}$$

Equation (16) is nothing more than equation (14) without the non-linear terms and with equation (15) subbed in. It is then simple to compare the size of the two terms on the right hand side of equation (14) to either term in (16) to known under what situations the non-linear terms may be neglected. Linearity is safe to assume when,

$$\left|\overline{u}-c_x\right| >> \left|u'\right|, \text{ (17a)}$$

and

$$\frac{\partial \overline{q}}{\partial y} >> \left|\frac{\partial q'}{\partial y}\right|. \text{ (17b)}$$

Recalling that the amplitude of linear vertically propagating Rossby waves increases exponentially with height (see equation 4) in an atmosphere under a constant mean-zonal wind field, there must be some level where the amplitude becomes so large that wavebreaking must occur. This occurs at the critical level, when the Doppler-shifted phase speed of the wave disappears, i.e. $\overline{u} - c_x = 0$. Under such conditions where the mean flow vanishes, 17a cannot be satisfied and the waves break. Below is a schematic that explains the different critical levels for varying Rossby wavenumbers. Notice how only the longest waves, wavenubers 1 and 2, have a critical level above the torpopause. Higher wavenumbers will undergo wavebreaking before penetrating the tropopause and become refracted toward the equatorial critical line like the wave rays do in figure 7. The thick black line represents the critical line, while the thin dashed line indicates the location of the tropopause. Each vertical thin black line indicates the relative height at which each wavenumber would encounter the critical line and hence undergo wavebreaking.

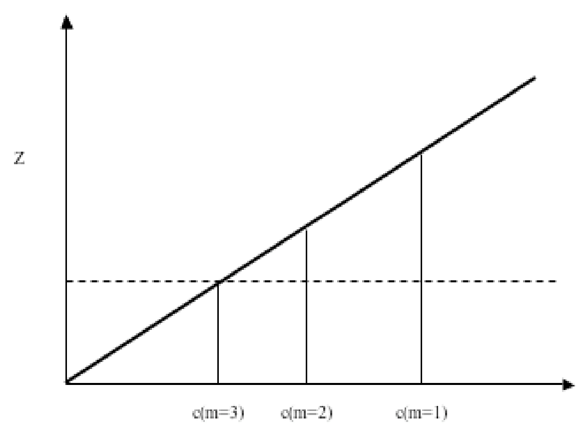


Figure 8 – Schematic depicting critical level for various Rossby wavenumbers (see text for details)

5. Sudden Stratospheric Warmings

Observational studies have confirmed that the vertical propagation of primarily topographically forced Rossby waves into the stratosphere is the cause for sudden warming events that occur in the stratosphere around midwinter about every other year. Holton (2004) chapter 12.4 describes such events as follows. During normal years, the mean temperature profile of the lower stratosphere has minimum temperatures above the equator with a temperature maximum at the summertime pole. In the wintertime hemisphere a local maximum around 45° exists followed by a rapid decrease in temperature toward the pole (see Holton figure 12.2). The rapid decrease is explained through the thermal wind requirement of a zonal vortex with strong westerly shear with height. About every other year a breakdown of the zonal vortex occurs due to enhanced vertical propagation of Rossby waves. The breakdown happens quickly in the span of a few days and leads to a reversal of the meridional temperature gradient and creation of a circumpolar easterly current due to a sudden warming of the stratosphere. An example of a sudden stratospheric warming event is given in figure 9 below. Normal conditions prevail in figure 9a. The vortex is slightly distorted from wavenumbers 1 and 2, which produce the Alutian High. Ten days later the wavenubmer 2 has increased in strength leading to a splitting of the polar vortex and increased warming over the North Atlantic and Russia, figure 9b. Five days later, the polar vortex has completely disappeared and temperatures in the polar region have increased by ~ 50°C, the meridional temperature gradient has reversed, and the mean zonal winds north of 65° have become easterly.

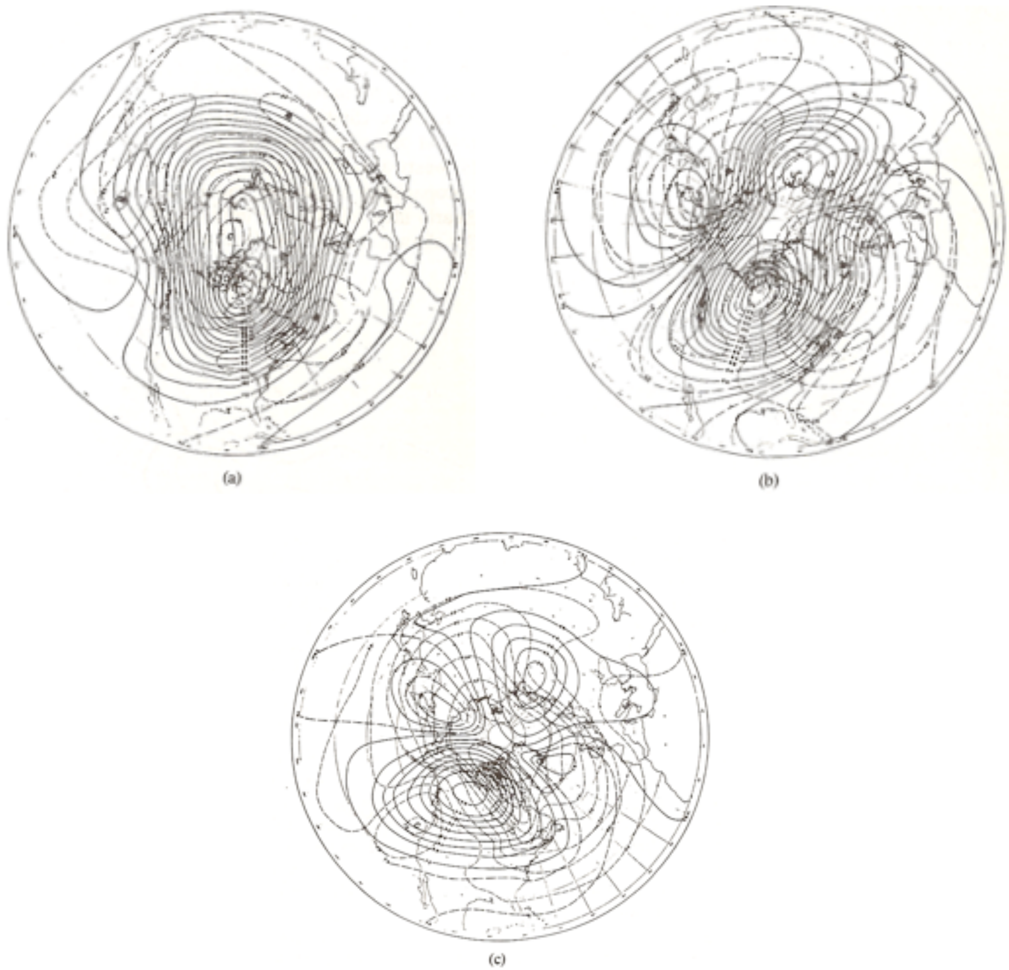


Figure 9 – Evolution of the 50-mb height field (solid contours in 100s of feet) and temperature field (dashed contours in °C) during January and February, 1957: (a) 25 January, (b) 4 February, (c) 9 February. (Holton 1975)

These sudden warming events occur through strong interaction of the waves and the mean flow of the stratosphere, associated with wave transience (i.e. change of amplitude with respect to time) and wave dissipation. The mean flow is decelerated through the amplification of the waves and propagation into the stratosphere, leading to a breakdown of the strong westerlies, allowing the formation of an easterly current. The details of this process are beyond the scope of this paper, as it deals with the Eliassen-Palm flux, which will be discussed in another paper.

6. Conclusion

This paper examined the phenomenon of vertically propagating Rossby waves. It was shown that vertical propagation of topographically forced Rossby wavenumbers 1 and 2 into the stratosphere occur predominantly in the wintertime northern hemisphere under weak westerly winds. During the wintertime the ‘polar night jet’ is able to act as a waveguide for the Rossby waves, allowing them to propagate high into the atmosphere. The strong westerly jet acts as a channel in which the waves get trapped and behave like rays as they propagate vertically. The vertical propagation becomes very important every other year or so to explain sudden stratospheric warming events. The waves are able to disturb the mean flow of the stratosphere and alter the meridional temperature gradient, thus leading to warming events.

7. References

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