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Tobias Wolf

1. Abstract

Planetary wave (PW) activity weakens the stratospheric south polar vortex. Therefore phenomena like polar stratospheric warmings can occur. In those cases the polar vortex becomes weakened strongly until it breaks down, e.g. reverses the vortex winds or splits it into two vortices.
The propagation behavior of planetary waves depends strongly on the velocity and direction of the background wind flow. Hence they should be affected strongly by the Quasi-Biennial Oscillation (QBO) which, even though it is a purely equatorial effect, influences the entire global wind field by affecting the transmission of atmospheric waves.
To find out if the PW activity at the south polar vortex is influenced by the QBO, large amounts of data were evaluated with Matlab:

- Quasi-Biennial Oscillation wind fields
- Temperature, ozone concentration, longitudinal wind velocity and the geopotential height at the south polar vortex
- Measurements of the solar activity i.e. the solar cycles

Through a correlation analysis, it was found that the south polar planetary waves are indeed influenced by the Quasi-Biennial Oscillation and that both might be influenced by other effects like the solar cycle or the lunar nutation.

Contents

2. Introduction and theory

2.1. Introduction

Earths atmosphere is mainly composed by Nitrogen (N_2) with a volume mixing ratio of about 78%, and Oxygen (O_2), that has a volume mixing ratio of about 21%. The remaining atmosphere is composed of so called minor species. The three most important ones are carbon dioxide (CO_2) -that traps the from the earth outgoing radiation-, water vapor (H_2O) and ozone (O_3) -both directly absorb incoming solar radiation-, with volume mixing ratios of about 0.036%, 0.03% and 0.001% [Andrews, 2000].

A non rotating even earth with a constant illumination all over the globe would cause a static atmosphere without any winds and without any weather since all incoming illumination goes into a uniform heating of the entire surface and the atmosphere.

But none of these assumptions are true. We have winds and waves driven by temperature gradients and a varying Coriolis coefficient. These can affect each other as well as modify the minor species that were responsible for the absorption of the solar energy in the first place.

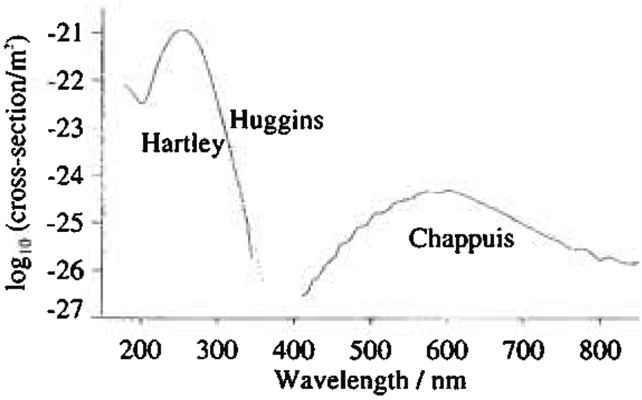
The motivation of that work is to extract the planetary wave activity and to figure out if and how it depends on the Quasi-Biennial Oscillation winds as an example of amplifying non steady state effects.

2.2. The steady state wind fields and atmospheres equilibrium

In this part will first be explained the most important minor species, ozone. Afterwards will be explicated the atmospheres equilibrium that is mainly caused by ozone and therefore by the suns influx. Finally will be illustrated the steady state wind fields, that again are caused by solar influx in different angles on the equilibrium atmosphere [Andrews, 2000].

2.2.1. Ozone

Ozone is the most important species in the atmosphere for the steady state wind fields and atmospheres equilibrium because of its high absorption rate of solar light in the ultra violet spectrum. The absorbtion spectrum for ozone is shown in figure (2-1).



2-1 absorption cross section for O3 [Andrews, 2000]

The maximum concentration of ozone in earths atmosphere is mostly found in a height of 25 km due to its production and destruction mechanisms [Andrews, 2000]. The region between a height of 20 and 50 km is named as ozone layer. The normal distribution of ozone is shown in figure (2-2)

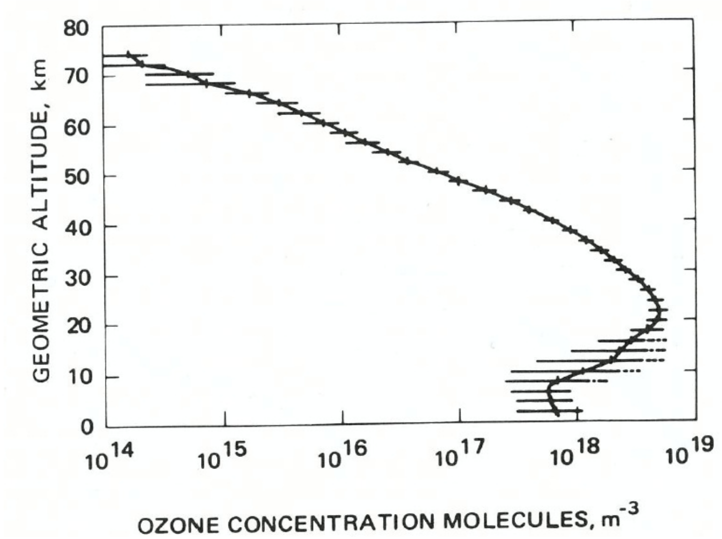


Figure 2-1 average ozone concentration over altitude for mitt latitudes [Andrews, 2000]

2.2.2. The atmospheres equilibrium

The earth's atmosphere has constant trends of temperature over long distances in altitude. The structure of the atmosphere, shown in figure (2-3) and the reasons for that structure are here briefly given in a list [Andrews, 2000]

- Troposphere (≈0 to 8-18km). Here we live, airplanes fly and most of the weather takes place. The temperature falls from the ground to the top, since almost the only warming is due to the heating of the ground. Temperatures fall down to (-45°C) – (-75°C) by 18 km.
- The tropopause is the boundary layer between the troposphere and the stratosphere.
- The stratosphere (≈8-18km up to 50km). Here, the temperature rises again due to the absorption of incoming solar ultraviolet radiation by ozone, which has the highest volume mixing ratio in this layer (compare to figure (2-2)). The temperatures rise up to 0°C again. Here, polar stratospheric clouds (PSC) can occur.
- The stratopause is the boundary layer between the stratosphere and the mesosphere.
- The mesosphere (≈50 to 85km). Here, almost no radiation is absorbed but the atmosphere continues to radiate away energy. Thus the temperatures fall again down to ≈-90°C. This is also the place where meteorites start to burn up.

The remaining higher atmospheric layers are not important for the course of this work and so will not be mentioned.

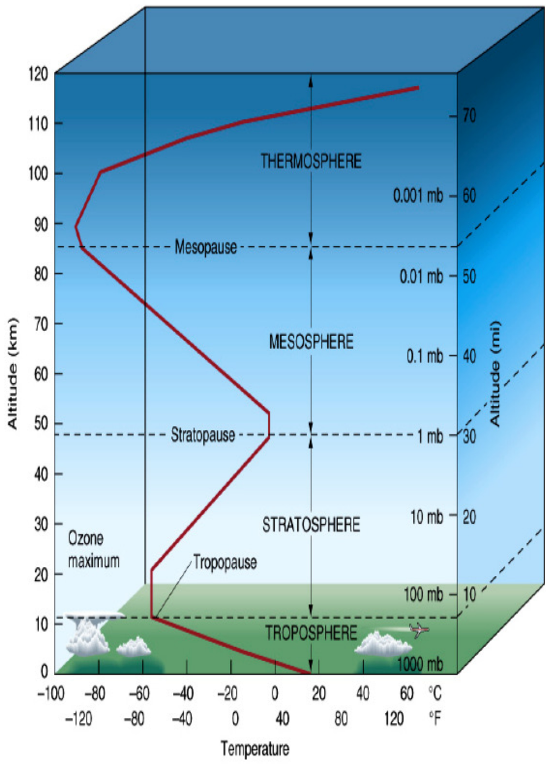


Figure 2-3 steady state of earths atmosphere with the division into layers. In the stratosphere is marked the maximum in ozone concentration

2.2.3. The wind fields

The incident angle of suns radiation on the earths atmosphere and ground varies from north to south and in one spot during the annual cycle. That causes differences in the temperature of the atmosphere and therewith differences in density between e.g. the equator and the poles for the same altitudinal levels. Winds that bring air from latitudes with higher pressure to latitudes with lower pressure attempt to correct these mass and therefore pressure differences.

Normally, the winds would blow straight in latitudinal direction, the direction of the temperature gradients (i.e. meridional winds). But since earth is spinning, a Coriolis force exists that deflects the winds and makes them rotate around the globe.

The strongest of these deflected winds blowing in longitudinal direction (i.e. zonal winds) is the winter polar vortex. It is strong enough to isolate the winter polar stratosphere from the rest of the globe. Therefore the temperatures in the polar stratosphere can fall extremely low, driving a mechanism that leads to a depletion of ozone causing the "polar ozone hole" [Brasseur, 1984]. In the wind field, given in figure (2-4), one can easily recognize the polar vortex at the winter pole as the strongest winds.

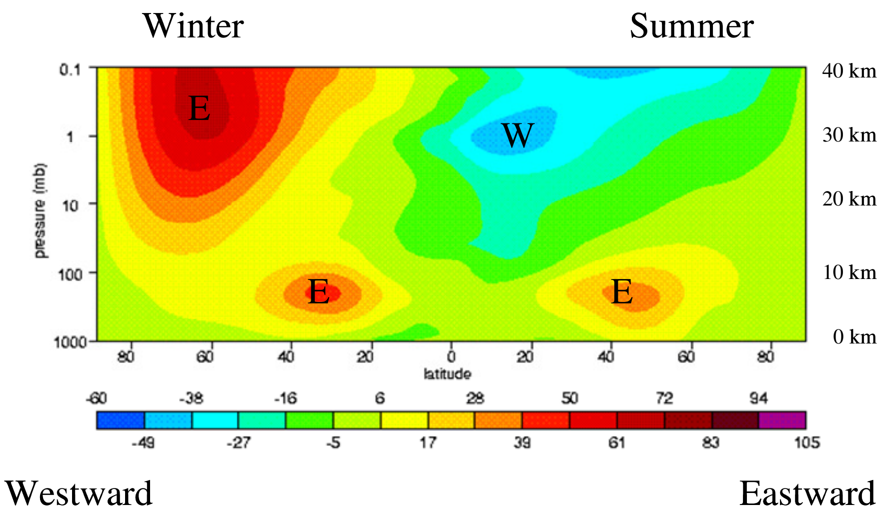


Figure 2-2 atmospherical wind field (without any wave activity) [Espy, 2009]

Since the main object of analysis for this work is the Antarctic winter vortex, here will be taken a closer look at it:

During the southern winter there is no solar influx on the South Pole. Consequently, the temperature starts to fall and so the atmospheric column shrinks. As an example, the altitude of the pressure level of 1 mbar drops from 50km to 40km at the pole. Therefore, the pressure in a height of 50km at the equator is higher than at the South Pole. Winds will arise and try to balance that difference in pressure at all altitudes.

The temperature gradients are strongest as one enters the dark polar regions. So the strongest winds will rise at the polar stratosphere since the most warming is missing here because stratospheric temperatures depend on the absorption of UV light by ozone. But those winds will be deflected strongly due to the strong Coriolis force, which increases in strength toward the poles, and therefore start to rotate eastwards (easterly winds).
That eastwards rotation now shields the polar stratosphere from the rest of the globe and avoids it warming up. Therefore can the temperatures fall further. They will first start to rise again when the polar vortex breaks down either by dynamic wave forcing, or the spring begins and sunlight warms the inner vortex up again, leading to a breakdown of the vortex [Brasseur, 1984].

2.3. Perturbations in the steady state wind fields (atmospheric waves)

In the last part was explained how Earth's atmosphere reaches a stable equilibrium fluid.
In principal, even not under the assumption of an even surface, this equilibrium tries to be kept, since the contours of Earth's surface are not big enough to destroy the entire pattern, but being able to perturb the pattern quite a bit, for example with waves. They can be defined as propagating disturbances whose acceleration is balanced by a restoring force [Brasseur, 1984].

2.4. Small scale waves (gravity waves)

Small scale or gravity waves are oscillations with relatively short horizontal wavelengths (typically 10 - 1000 km) that arise in a stably stratified fluid when air parcels are displaced [Brasseur, 1984].These displacements can be due to e.g. mountains or thunder storm systems that accelerate the equilibrium winds in vertical direction when they pass them. The restoring force in that case is the buoyancy of the stable equilibrium atmosphere. The propagation of gravity waves depends strongly on the atmospheric wind structure. For example a wind in the same direction with the same velocity as the phase velocity of the wave can absorb that wave. The point where this occurs is referred to as the critical level [Brasseur, 1984]. Figure (2-5) shows the propagation of gravity waves under the influence of a background wind field.
The waves can propagate freely until they either (a) reach a wind with the same or higher velocity as their phase velocity, (b) break when they reach environments for which non-linear effects can no longer be neglected [Brasseur, 1984], or (c) reach the dissipation point, where the amplitude falls and the momentum gets transferred into thermal motion.

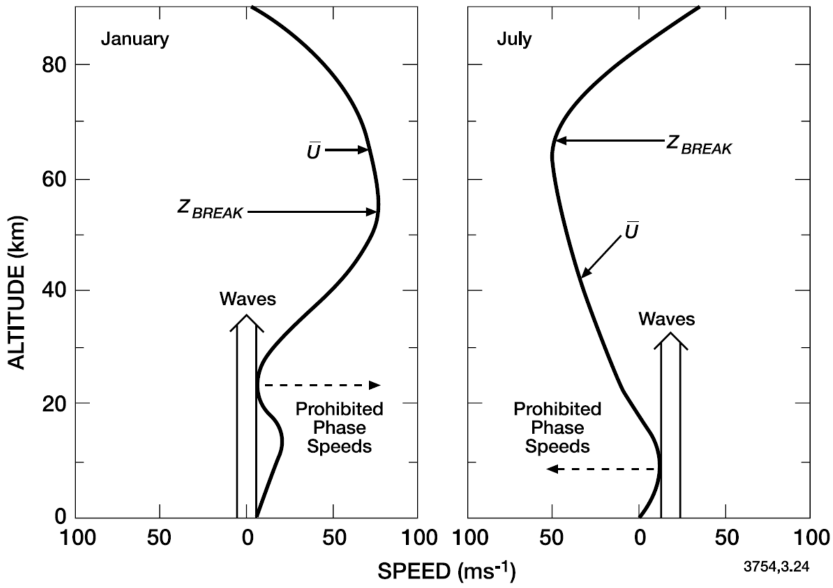


Figure 2-3 effect of the wind fields (e.g. the steady state wind fields) on the propagation of gravity waves [Brasseur and Salomon, 1984].

2.5. <u>Large scale waves (planetary waves)</u>

This section is based on the chapter "Further Atmospheric Fluid Dynamics" of the book written by David G. Andrews [Andrews, 2000].

Planetary waves are another kind of wave that disturbs the equilibrium atmosphere. They are global scale, horizontal, transverse atmospheric waves with a westwards directed propagation. The origins of planetary waves are displacements in the longitudinally propagating meridional winds in the atmosphere in horizontal direction. In this case the restoring force is the potential vorticity. The wave number of planetary waves, that is the number of cycles around a circle of longitude, generally reaches from 1 to 10.

2.5.1. The potential vorticity

Potential vorticity is a combination of the vorticity in the frame of reference and the vorticity of the frame of reference i.e. the Coriolis parameter (remember: the Coriolis parameter increases from the equator to the poles). The potential vorticity is a conserved parameter and creates a restoring force on each air parcel.

Vorticity is an expression for the shearing strain of flows and is best explainable in a hypothetical vorticity meter schematically given in figure (2-6). The vorticity meter consists of 4 perpendicular vanes. One of these vanes is marked with a black dot that makes it distinguishable from the others, so potential rotations of that vane

can be observed. If one takes now that vane and puts it in a solid body rotation as in figure (2-7) or a rectilinear shear flow as in figure (2-8) one can observe different behavior of rotating of the vorticity meter:
In the solid body rotation, the black spot stays always facing from the center of the rotation, because the circle at the outer vane rotates with the same angular frequency and therefore with higher tangential velocity than the circle at the inner vane. So the vorticity meter performs, additionally to the cycle around the center of the solid state rotation, another rotation around its own axis.
In the shear flow the vorticity meter moves parallel to the flow, but since the lower jet moves more slowly than the upper jet, it rotates always once. That is when the difference in distance moved by the upper stream and the lower stream is exactly the circumference of a circle with the center at the center of the vorticity meter and with diameter of the length of a perpendicular line between the two jets. Closer examinations can show that the vorticity at the vorticity meter is twice the rotation frequency of the vorticity meter. So the vorticity is a measurement of velocity gradients in constant flows.

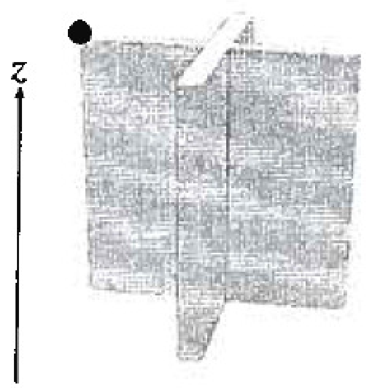


Figure 2-4 theoretical vorticity meter [Andrews, 2000]

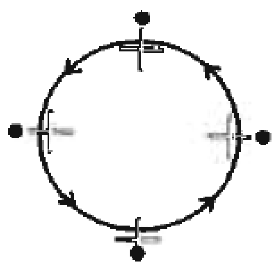


Figure 2-5 vorticity meter in a solid state rotation [Andrews, 2000]

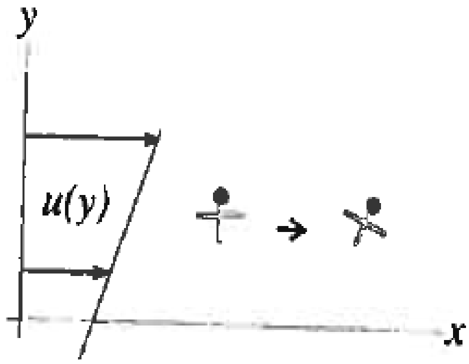


Figure 2-6 vorticity meter in a 2-dimensional shear flow [Andrews, 2000]

2.5.2. How does the Potential vorticity cause a planetary wave?

Figure (2-9) illustrates the mechanism that builds planetary waves for the case of the northern hemisphere:
A line of air parcels are lying on a line of latitude. Now these air parcels get displaced in a sinusoidal pattern by e.g. gravity waves [Smith, 2003] (solid line). Parcel A moves southwards and so the vorticity of the frame of reference shrinks. The air parcel spins up in the direction the Coriolis force drives it, namely anticlockwise.
In addition, viscous friction lets the local velocity fields spin up, which moves parcel B.

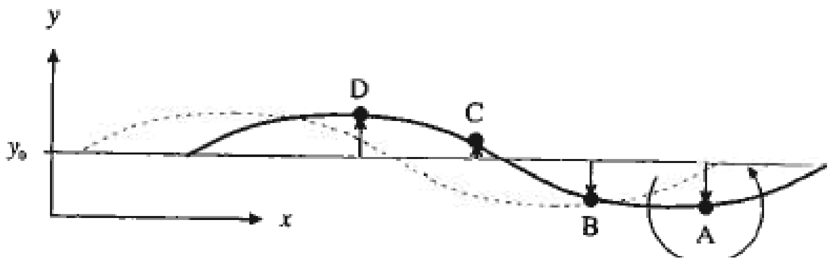


Figure 2-7 simplified 2-dimensional PW mechanism, in terms of conservation of potential vorticity [Andrews, 2000]

The velocity fields around B spin up, making parcel C move southwards and parcel A move northwards again. Applying this argument to all the parcels after a short time, the pattern will have changed to the dashed line. So the entire pattern has moved westwards, even though the single air parcels only move north- and southwards.
A westward moving PW pattern emerges. As a next step, the pattern (reaching around the line of latitude once) amplifies due to the background flow. This makes the circulating air parcels even circulate faster (this is an example in large scale of the solid state rotation from above and in a small scale of the 2-dimensional shear flow). So the amplitude of the wave grows until a drop of cold air from the north detaches as in figure (2-10).
The wave amplitude shrinks again due to that loss of momentum and another cycle of a growing PW can start.

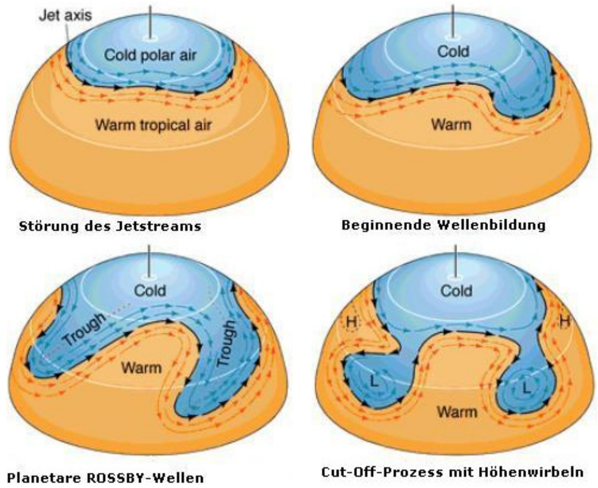


Figure 2-8 an amplifying PW with a detaching parcel of cold air [Boljahn, 2009]

Until now, the description was only stationary for one line of latitude. Of course, if one moves single parcels of air, it will affect the pattern in north-south direction as it affects the vertical pattern. What happens is that the wave not only propagates westwards, but it also propagates in the north-south direction and vertically.

In the north-south direction, it will propagate until it meets a wind structure that makes it break. That would either be wind in the same direction and speed as the phase velocity goes, or to strong of a wind against the propagation direction In these cases the wind makes it lose too much energy and disappear or break. That is basically the same effect as it happens with the gravity waves explained above, only with lower phase velocities.

Propagating in the vertical direction, it tries to keep its momentum and therefore grows in amplitude, since the density of the air shrinks rising in altitude, until it again either meets a wind structure that makes it break or disappear, or it can propagate freely and grow in amplitude until the wave amplitude becomes too large and breaks due to non adiabatic behavior (compare to water waves at the beach). Waves could also break when they reach the dissipation point, where the mean free path is so large that the air parcels dissipate before they can complete a wave cycle. The amplitude falls down and the momentum and energy is taken into thermal motion of the atmosphere.

2.5.3. The effect of PW in the global wind structure (especially the polar vortex)

PW have a special influence on the global wind structure in the winter stratosphere since they propagate in north-western or south-western direction over very long distances. Because of that they can even reach the winter polar circle with its polar vortex. Here, they encounter conditions in the wind fields that make them break. The eastern winds of the polar vortex increase until they reach a maximum at a height above 1 hPa (one can measure the altitude in either meters or due to the falling pressure in higher altitudes as well in hPa, but there is no direct relation, the pressure in a certain height depends on e.g. the temperature of the column underneath. Therefore, in many figures both are given). When PW propagate upwards, they break either due to non adiabatic behavior because of their increasing amplitude in the thinning air or due to the strong winds facing them. In any case they transfer momentum to the vortex wind. Since that momentum has the direction against the momentum of the polar vortex – slow the winds down or even reverse the polar vortex.

That breaks the shielding between the very cold polar air and the warmer air outside the vortex and causes an event called stratospheric warming. In the northern hemisphere, the water-land structure benefits the PW more than in the southern hemisphere, and strong polar warming events occur over the Arctic regions nearly every year. However, only one strong polar warming event over Antarctica has been detected in 2002 [Greenbelt, 2002] and it is still not clear why the PW were so strong that year.

PW do not reach breaking altitudes in the summer stratosphere, since during the summer they encounter strong westward directed mean flows at low altitudes and do not propagate.

2.6. <u>The Quasi-Biennial Oscillation (QBO)</u>

This chapter is based on the paper by Baldwin [Baldwin, 2001].

The QBO is a systematic oscillation in the equatorial zonal steady-state winds that is caused by gravity wave interaction with the mean flow. This can influence both, gravity waves and planetary waves, since their propagation depends on background wind velocities.

2.6.1. The discovery of the QBO

In 1883, the debris from the Krakatau eruption cycled around the world in about 2 weeks: the “Krakatau easterlies”. However, in 1908, the meteorologist A. Berson launched balloons in the tropics and found westerly winds regimes: the “Berson westerlies” [Hamilton, 1998]. Due to the irregularity of balloon soundings it was not until 1959 when a periodical wind pattern was observed: The wind speed above Christmas Island (2.0°N) showed gradually descending easterly and westerly wind regimes [Graystone 1959]. Finally in 1960 R.J. Reed from the US used rawinsonde data at Canton Island (2.8°S) and found “alternate bands of easterly and westerly winds, which originate above 30km and which move downward through the stratosphere at a speed of about 1km per month.” [Reed, 1961].

Independently but at the same time, Ebdon used data from Canton Island spanning 1954 to 1959 [Ebdon, 1960]. He also found a period of about 26 months in agreement with Reed. In a later work, Ebdon found -using data from 1954 to 1960- a period of 26 to 27 moths [Ebdon, 1961] using data from 1954 to 1960.

To describe this, the name Quasi-Biennial Oscillation was first used by Angell and Korshower in 1964 in their work “Quasi-Biennial Variations in Temperature, Total Ozone, and Tropopause Height“[Angell, 1964].

2.6.2. The main properties of the QBO

The QBO is a large scale oscillation of winds at the Equator with a period of approximately 28 months. That means about every 14 months the wind in the lower stratosphere turns from westwards to eastwards or vice versa. One important aspect is that this oscillation propagates downward in about 14 months without any loss of amplitude. Normally, waves that propagate downwards should lose amplitude due to the atmospheres higher density in the lower altitude regions [Brasseur, 1984]. Even though the QBO is not an true biennial oscillation, it has a clear seasonal preference as it can be seen in figure (2-11).

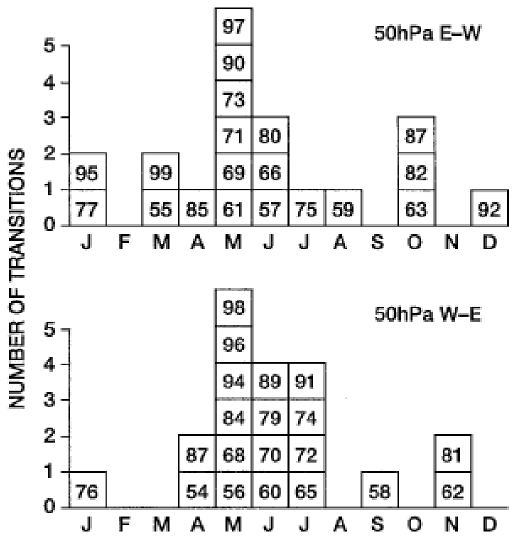


Figure 2-9 histograms of the number of zero crossings at 50 hPa grouped by month

In figure (2-12) a dynamical overview of the QBO during northern winter is given. The propagation of various tropical waves is depicted by orange arrows, with the QBO driven by upward propagating gravity, inertia-gravity, Kelvin, and Rossby-gravity waves. The propagation of planetary-scale waves (purple arrows) is shown at middle to high latitudes. Black contours indicate the difference in zonal-mean zonal winds between easterly and westerly phases of the QBO, where the QBO phase is defined by the 40-hPa equatorial wind. Easterly anomalies are light blue, and westerly anomalies are pink. In the tropics the contours are similar to the observed wind values when the QBO is easterly. The mesospheric QBO (MQBO) is shown above ~80 km, while wind contours between ~50 and 80 km are dashed due to observational uncertainty [Baldwin, 2001].

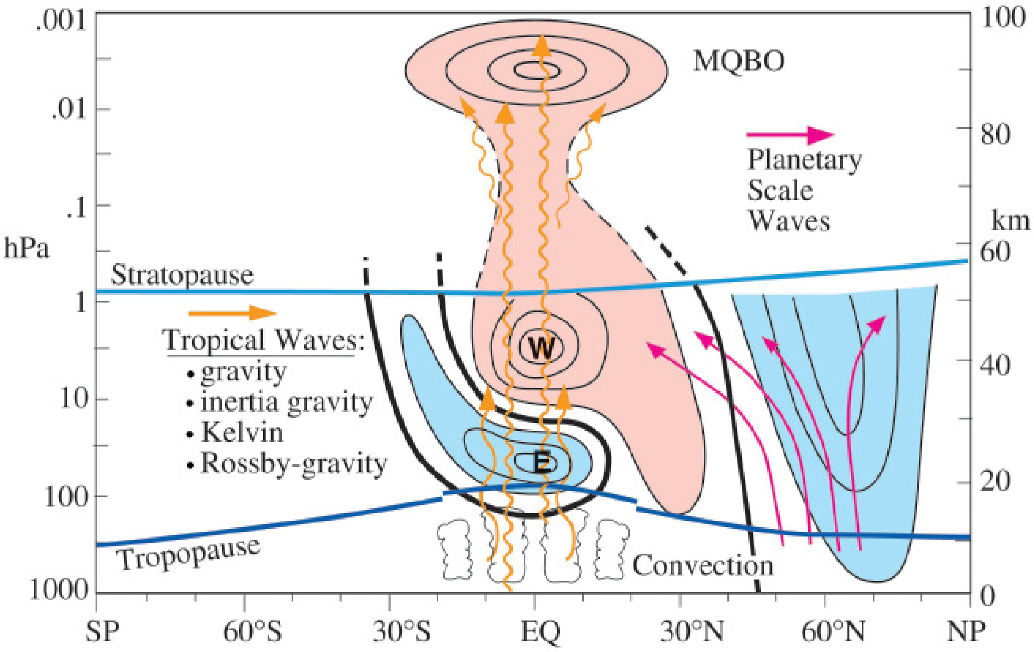


Figure 2-10 dynamical overview of the QBO during northern winter

The most remarkable features of the QBO, that any theory must explain, are

(1) the quasi-biennial periodicity

(2) the occurrence of zonal symmetric westerly winds at the equator since conservation of angular momentum does not allow zonal-mean westerly advection to create an equatorial westerly wind maximum [Baldwin, 2001], and

(3) the downward propagation without any loss of amplitude

Over the time there have been several theories describing how the QBO is driven. Wallace and Holton tried to drive it in a numerical model through heat sources or through extra tropical planetary scale waves [Wallace, 1986] but they showed rather conclusively that lateral momentum transfer by planetary waves could not explain the downward propagation of the QBO. However, they realized, that the only possible explanation for the downward propagation is a downward propagating driving force.

Lindzen found that vertically propagating gravity waves could provide the necessary wave forcing for the QBO. He and Holton showed in a two-dimensional model how a QBO could be driven by a broad spectrum of vertically propagating gravity waves [Lindzen, 1968]. The existence of these waves was only an assumption, but the model worked as Plumb [Plumb, 1978] showed in a laboratory experiment, using a large annulus with some special properties, as they found clockwise and counterclockwise propagating gravity waves. Thereby, the fact that the QBO has almost the same period as a sub harmonic of the annual cycle is pure coincidence.

2.6.3. The driving mechanism for the QBO

In 1978, Plumb explained the mechanism that explains the features of the QBO as interaction between small scale (gravity) waves [Plumb, 1978]. Figure (2-13) explains the mechanism, where the waves driving the QBO wind oscillation are shown by orange arrows (eastward waves) and blue arrows (westward waves):

a) For most of the westward waves the wind speed is higher than the wave phase velocity above a certain altitude. Therefore the waves reach a level of critical wind speed and transfer their momentum and energy to the mean flow. The waves that do not reach a critical wind speed propagate upwards until they break due to non adiabatic behavior. Another category of waves has phase speeds barely bigger than the wind speed. They transfer energy and momentum to the wind via viscous friction, but only over a vey narrow range. All three mechanisms drag the westward wind phase down. The same process occurs for the eastward waves shown in the figure. Finally, the winds with eastward phase will be pulled down so far that all waves break and none can propagate.

b) The eastward winds were dragged down so far that the layer became too narrow and the shears too high to be sustained. Hence, the eastward wind dissipated. The eastward propagating waves can now propagate freely and give their entire momentum by breaking and viscous friction to the winds at high altitudes, where they produce a new eastward phase. The westward propagating waves continue to reach critical levels at the westward winds at lower altitudes, continuing to drag the westward wind phase down.

c) A new eastward wind phase was produced at high altitudes and the waves with low eastward phase velocity begin to reach critical levels when their phase speed equals the wind speed again. Here they transfer their entire eastward momentum to the wind. Furthermore, the waves with higher eastward phase velocity continue to propagate to high altitudes and lose their momentum by breaking and viscous friction. They continue to drag the eastward wind phase down, as do the westward propagating waves with the westward wind phase. These westward waves create a strong, thin westward wind profile that becomes susceptible to shear forces and will eventually dissipate.

d) – (f) the cycle starts again, but with opposite wind phases.

This mechanism can even start a QBO, since only a small displacement in a constant wind field is required and the cycle can start either from (b) or from (e), depending on the direction of the displacement. These displacements can for example be large scale waves that propagate until the equator.

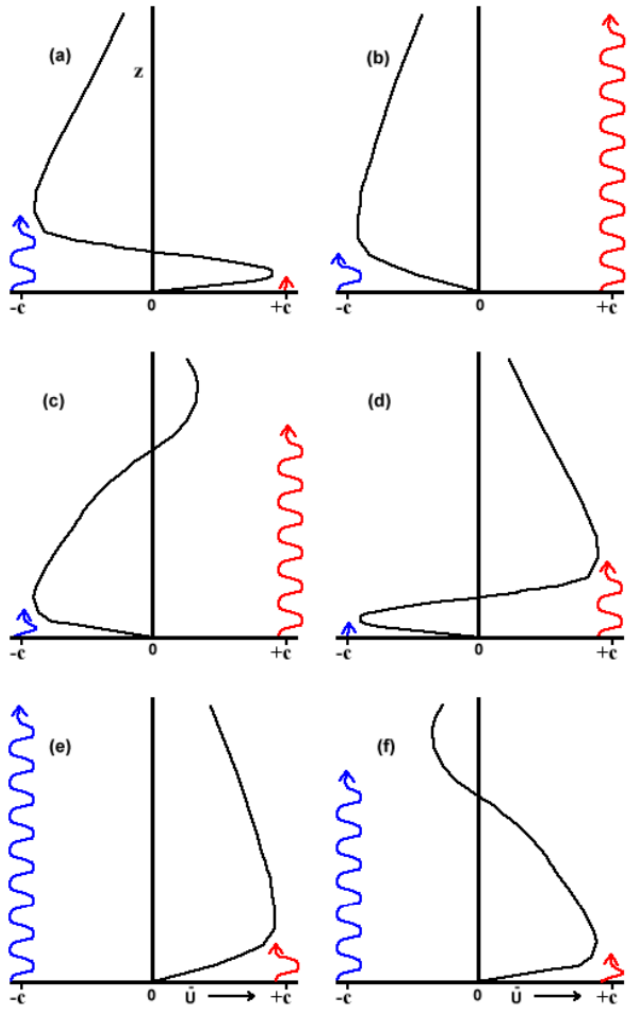


Figure 2-11 mechanism for the QBO [Plumb, 1977; Heaps, 2009]

2.7. <u>Solar Cycle</u>

Until now the steady state of the atmosphere and effects that perturb the steady state were explained, but the most important factor for all of these effects was not covered: The suns activity.

Over very long time scales, perhaps millions of years, the suns activity is relatively constant, but seen in shorter time scales on the order of tens of years, it can vary. One of these variations is the solar cycle.

2.7.1. The solar cycle

The solar cycle is a periodical variation in suns activity due to the reversal of its magnetic field with a period of about 11 years. This variation varies the influx of solar energy onto the earth as well. Thereby, it could rise temperatures and enhance effects like the atmospheric winds (e.g. polar vortex, QBO).

The reversal of the suns magnetic field happens because the equator rotates faster than the poles. Thereby the field lines become rapped around the globe. The magnetic field strength becomes so high that the field lines break and reconnect outside the surface of the sun (compare to the dynamo theory [Stix, 1971]).

Where the field lines penetrate the surface of the sun there is an enhanced field that blocks the normal convection of hot material up from below. Thus, the spot itself is cooler than the unperturbed surface. Figures (2-14) and (2-15) show pictures of sunspots. However, one gets enhanced convection at the edges of the spot that brings up the hot material more effectively. Thus the edges (faculae) are hotter and brighter than the unperturbed surface. The integrated effect is that the intensity enhancement form the faculae is greater than the darkening from the spots, and the sun is brighter at solar max (by 0.1% in the visible, about 2x brighter in the UV).

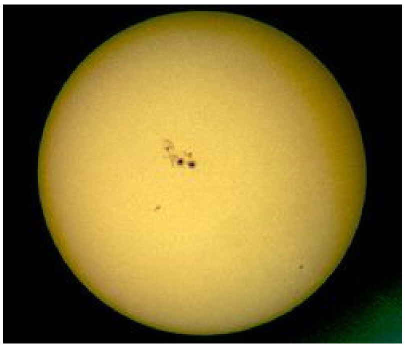


Figure 2-12 sunspot (dark spot in the middle of the picture) [Wikipedia, 2004]

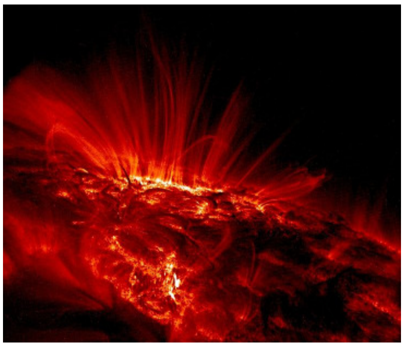


Figure 2-13 sunspot viewed in ultraviolet light [Nemiroff, 2002]

The period is about 11 years, but can vary strongly, as one can see right now (2009), where the activity of the sun should have been rising again, but still stays close to 0.

The amplitude of the maximum activity varies as well as can be seen in figure (2-16) which shows the F10.7 radio flux (compare to "The f10.7 radio flux" in the chapter Materials and Methods). This radio flux reflects the level of solar activity and shows the variation from 1953 to 2002.

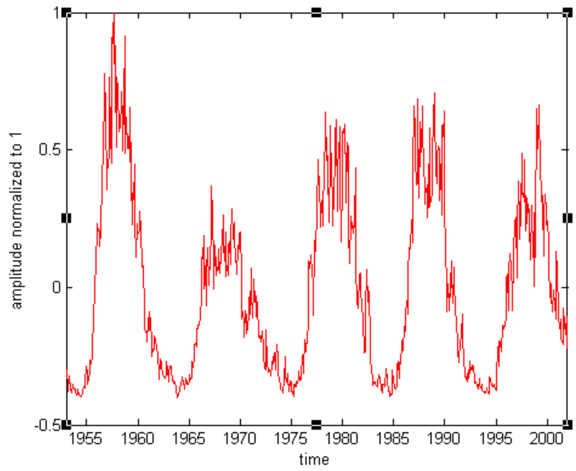


Figure 2-14 the monthly averaged F10.7 radio flux

2.8. Hypothesis

As said before: the Quasi-Biennial Oscillation may influence the planetary wave activity due to change of the background wind flow. The hypothesis that this project tries to answer is therefore:

The Quasi-Biennial Oscillation should block the planetary waves from propagating equatorwards, in the case when the phase of the QBO is westward. This blocking and reflecting under westward QBO phase should show up in the data as a maximum of the planetary wave activity near the poles.

Earlier projects tried to answer this question by comparing the polar vortex winds to the Quasi-Biennial Oscillation. In this project will be tried to use planetary wave amplitudes extracted out of the mean flow to get a more significant verification of the mechanism.

3. Materials and Methods

3.1. Introduction

A broad spectrum of analyzing tools was and data used and developed during the course of this project. To give the reader a short introduction into the parts "The analyzing tools" and "The datasets" were written. More detailed information about each tool or dataset can be found in the citations.

3.2. The analyzing tools

3.2.1. The Matlab cross-covariance function

The Matlab cross-covariance function (xcov) gives back an estimate of how similar the deviations form the mean of two datasets are to each other. The algorithm follows the form

$$x\mathrm{cov}_k(X,Y) = \sum_j (x_j - \mu(X)) * (y_{j+k} - \mu(Y)) \qquad (3\text{-}1)$$

where k is the lag between X and Y and $\mu(X)$ is the average over X and j is the number of the value in the dataset.

It does basically the same as the cross-correlation function (xcorr), but it first takes off the mean from the datasets before multiplication and calculating. Furthermore the results have been normalized. That was done to achieve a comparability to the coefficients of linear corellation. Thus the normalization is between 1 and -1 such that the auto-covariances at zero lag are 1.0.

3.2.2. The Levenberg-Marquardt fitting algorithm (LMF)

During the data evaluation the LMF [Balda, 2007], a nonlinear least squares fitting algorithm, was used several times. For example, it is used for fitting combined cosine and sine waves with different wavelengths to the datasets to find out the amplitudes of the planetary waves.

The algorithm calculates the least square fit between a given dataset (X) and a given function (Y) and gives back the best fit parameters of that function in a separate array (for example, fitting the function

$$\mathrm{Y} = A * \cos(B) \qquad (3\text{-}2)$$

The function will return an array consisting of the best fit values of A for all B).

The problem with this function is that if the values in the dataset become too big it will fail. Therefore, the data first need to be divided by an appropriate factor and after reconstruction multiplied with that factor again. A good idea is as well to check the significance of the fit. This can easily be done since the program gives back the sum of squares of the residuals of the fit (ssq). If

$$\text{ssq}/\sqrt{\sum_i X_i^2} > 0.01 \qquad (3\text{-}3)$$

the algorithm failed and a warning appears.

3.2.3. The Lomb-Scargle periodigram

To check the confidence level of peaks resulting from the Fourier transform in Matlab, the Fastlomb [Saragiotis, 2008] [Press, 2001] function was used. This routine least-square fits individual sine waves to the dataset to be analyzed. It returns the frequency spectra and additionally the statistical significance for each periodic component in the spectra.

3.2.4. The netCDF reader

The used ERA 40 dataset is retrieved as a Network Common Data Format file (.cdf), which needs to be read into Matlab. Therefore a netCDF reader [Spencer, 2007] was used. It simply reads in the netCDF files and puts them into Matlab arrays.

3.2.5. The Matlab fft function

To analyze datasets in terms of the frequencies contained the Matlab Fast Fourier Transform (fft) function was often used. It computes the frequency spectra of the discrete Fourier transform.

Problems came up when it was tried to find out the coupling between two cosine waves with different period [Chu, 2000] by computing the cross spectra. If two waves of equal amplitude but different periods couple probably with an unknown coupling of the form

$$\left[\cos(\frac{2\pi*t}{a})\circ\cos(\frac{2\pi*t}{b})\right]\otimes\left[\cos(\frac{2\pi*t}{a})\circ\cos(\frac{2\pi*t}{b})\right] \qquad (3\text{-}4).$$

Here $\circ$ is either addition or multiplication, the wavelength a>>b, the related frequencies f(a)<<f(b) and the cross covariance $\otimes$.

In this case it was not clear which structure the cross spectra would have under the two different couplings of multiplication and addition. In some cases both peaks might not appear as separate peaks. Instead they could be

split or even not show up at all. Therefore it was needed to produce examples with all combinations of addition or multiplication to compare with the results of those obtained from the cross spectra in the chapter "Data evaluation and results".
The results were: If one Fourier transforms a function of the form equation (3-4) peaks will not always visible at both frequencies in the Fourier Spectrum. Instead under multiplication, the peak of frequency f(b) splits into two peaks with difference

$$\frac{1}{f_m} = \frac{1}{f(a)} = \frac{2}{f_u - f_l} \qquad \text{(3-5)},$$

where f_m (modulation frequency) is the frequency of the longer period sinusoid that modulates the faster oscillation i.e. $f(a)$, f_u is the peak at the higher frequency of the two split peaks, f_l the peak at the lower frequency of the two split peaks. The peak at f(a) disappears.

In the appendix are given figures (3-4) to (3-8). They are examples for Fourier Transforms from equation (3-4) with a=144 and b=12, performed with the Matlab fastlomb [Saragiotis, 2008] function.
Whenever a multiplicative coupling exists, the peak at the bigger frequency gets split into two peaks as one can see in figures (3-4), (3-5), (3-7), (3-8). Under a mixture of multiplicative and additive couplings as in figure (3-7) one can see, that the smaller frequency peak can be seen and even split as well. Under purely additive coupling as in figure (3-6) both peaks show up and neither of them is split. Furthermore one can see that under the mixture of couplings (one + and one $*$) the amplitude of the cross covariance becomes small (<<0.015) even in cases when no noise exists.

A problem that might arise is that a sinusoid sampled close to half its frequency will appear to be heavily modulated in the time domain. This is shown in figure (3-1) where a synthetic signal at a period of 2.3 years, which is approximately the QBO period, is sampled every year.
The data appear to be strongly modulated with a period of about 10 years, which is close to a solar cycle. However the power spectra of that shows only a single peak with no splitting as shown in figure (3-2) This demonstrates that while a "beat" frequency between the sinusoid and the sample frequency may appear in the time domain, it will not affect the power spectrum in the frequency domain.
So all interpretation of modulations will be done in the frequency domain rather than the time domain.

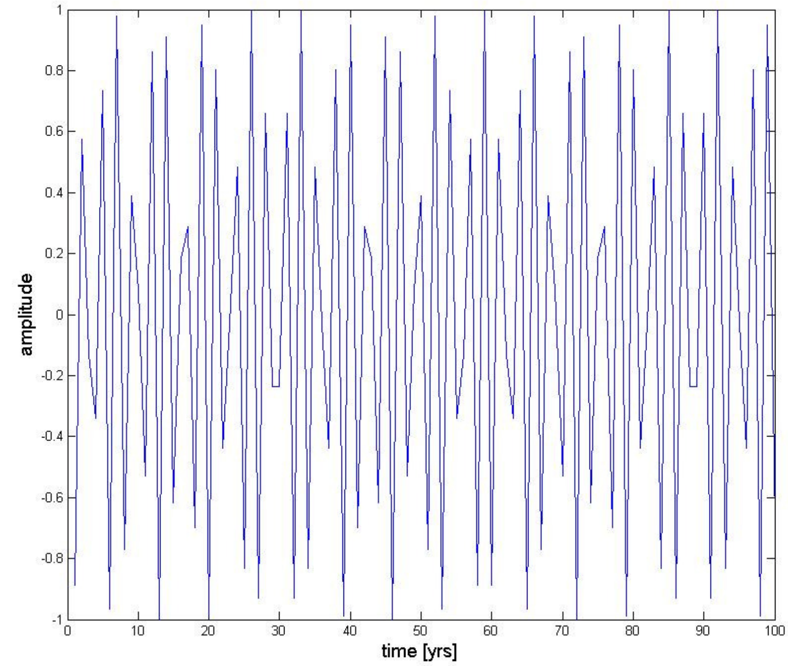


Figure 3-1 one wave with period close to the period pf the QBO, as one can see comes up a beating frequency even with when one wave is existing

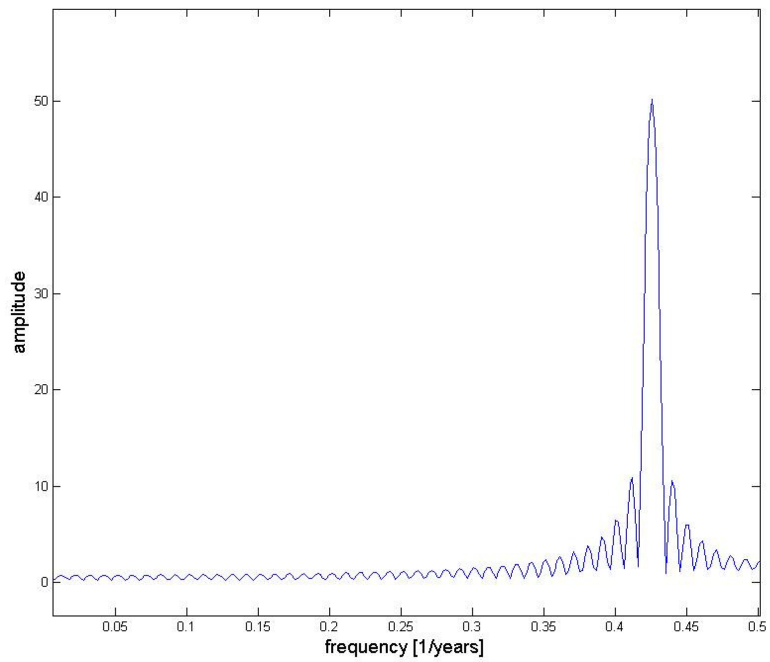


Figure 3-2 FFT of the data plotted in figure (3-1). There is only one single peak existing, so the upcoming beating frequency is only visible in the plot and no modulation

3.2.6. The Matlab Apodization function

To reduce end effects in the time domain to cancel out noise in fft and reduce the side lobe behavior of the transformed peaks, sometimes was used an apodization function: The Matlab Hamming function. It multiplies the data with a Gauss curve to taper the ends of the dataset smoothly to zero to avoid large data jumps when the data start with large and ends with small values. In this case, the fft function sees high frequency content in the rapid jump and creates multiple spurious frequencies that appear as noise in an attempt to duplicate it.

Furthermore the data were padded to at least 4 times the length and to a length that is a power of two. This avoids the fft from blending the beginning of the dataset with its end. That is needed since the fft function assumes the dataset to be periodic [Orfanidis, 1995].

3.3. The datasets

The ERA40 re-analysis

The ERA40 re-analysis [Uppala et al., 2005] was completed in 2003 by the European Centre for Medium-Range Weather Forecasts (ECWMF) providing a rich assembling of climate data during the period from September 1957 to August 2002, e.g. temperature data, wind fields, ocean waves, etc. It is an assimilation of climate data collected from data sources at random places which were assimilated in a computer general circulation model to give results for equally spaced data points with a 2.5° grid resolution 4 times a day. Obviously the more observed data points exist the more precisely the results of the model can be.

Table (3-1) shows the different data sources and their daily average number of data points. As one can see, the number of higher altitude operating systems as satellites has risen over the time.This is an important fact since the emphasis of this work is the area of the atmosphere between 10 and 100 hPa over Antarctica (between 40°S and 60°S) which is a huge area that is not coverable by e.g. balloon soundings. Since the data provided by the model reach over the entire planet, they had to be shortened to the needed spatial dimensions to avoid download limitations.

Observation type	1958-66	1967-72	1973-78	1979-90	1991-2001
SYNOP/SHIP	15313	16615	18187	33902	37094
Radiosondes	1821	2650	3341	2274	1456
Pilot balloons	679	164	1721	606	676
Aircraft	58	79	1544	4085	26341
Buoys	0	1	69	1462	3991
Satellite radiances	0	6	35069	131209	181214
Satellite winds	0	0	61	6598	45671
Scatterometer	0	0	0	0	7571
PAOBs	0	14	1031	297	277

Table 3-1 Average Daily counts of various types of observations supplied to the ERA-40 data assimilation, for five selected periods [Uppala, 2005]

The downloaded variables were:

The July data from 1971 to 2002 for

- the zonal wind velocity in longitudinal direction (u) [$\frac{m}{s}$]
- the Ozone mass mixing ratio (o3) [$\frac{kg}{kg}$]
- the (air) temperature (t) [K]
- the atmospherical geopotential height $\frac{m^2}{s^2}$

at the levels of (1, 10, 50, 100) hPa. This covered almost the entire stratosphere. That is the main area of the polar vortex and about the peak of maximum ozone concentration.

The times are chosen because July is mid winter where planetary waves should be strongest [Pancheva 2004]. The data from 1970 were used as satellite measurements, which cover entire Antarctica and other remote locations, were more available after this time. The data were achieved under pressure levels [Uppala et al., 2005] and were written as .cdf files.

3.3.1. The F 10.7 radio flux

The used data for the solar cycle is the F 10.7 [NOAA/WDC] which is a daily measurement of the 10.7cm radio flux from the sun. The 10.7 cm wavelength is near the peak of sun's radio emission. It is easily detectable from earth even under clouds and so provides the longest direct record of solar data (since 1947) available. The F10.7 radio flux is an indirect measurement of daily sun activity and therefore solar cycles.

Figure (3-3) shows the F10.7 flux, shortened to the length of the data sets to be analyzed, and averaged over each month with normalized amplitude.

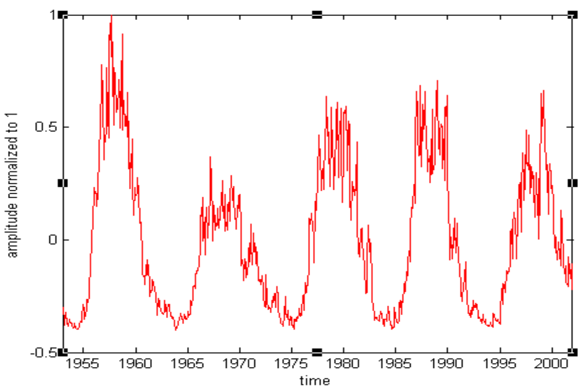


Figure 3-3 the F10.7 radio flux monthly average

3.3.2. The QBO data

For including the QBO data in climate chemistry experiments Giorgetta [2005] assembled and expanded a catalogue of QBO data brought together by B. Naujokat of the Free University Berlin [Naujokat, 1986; Labitzke et al., 2002]. Table (3-2) shows the origin of the dataset.

Giorgetta took Naujokats data and expanded them in time from 1953-2005 and in altitude to a total of 19 levels in hPa: 90, 80, 70, 60, 50, 45, 40, 35, 30, 25, 20, 15, 12, 10, 8, 6, 5, 4 and 3.

These data were used during this project to compare them to the PW amplitudes and the solar cycles. They are collected by rawinsonde measurements close to the equator. The definition of the phase of the QBO in terms of westerly or easterly is mostly according to the wind directions at 40hPa. Positive winds denote eastward phase (also known as westerly or winds out of the west), and negative winds denote westward phase (or easterly, winds, out of the east).

Station	***Coordinates***	***Months***
Canton Island (91700)	**02 46 S / 171 43 W**	**Jan.1953-Aug.1967**
Gan/Maledives (43599)	**00 41 S / 73 09 E**	**Sept.1967-Dec.1975**
Singapore (48698)	**01 22 N / 103 55 E**	**Jan.1976-Dec.2004**

Table 3-2 measurement stations of the QBO winds [Gioretta, 2005]

3.4. Appendix 3

Examples for the different cases of equation (3-4) with a=144 and b=12 $\begin{array}{l}\mathbf{X = 10 * \cos(2 * \pi * t/12)} \\ \mathbf{Y = 10 * \cos(2 * \pi * Z/144)}\end{array}$.

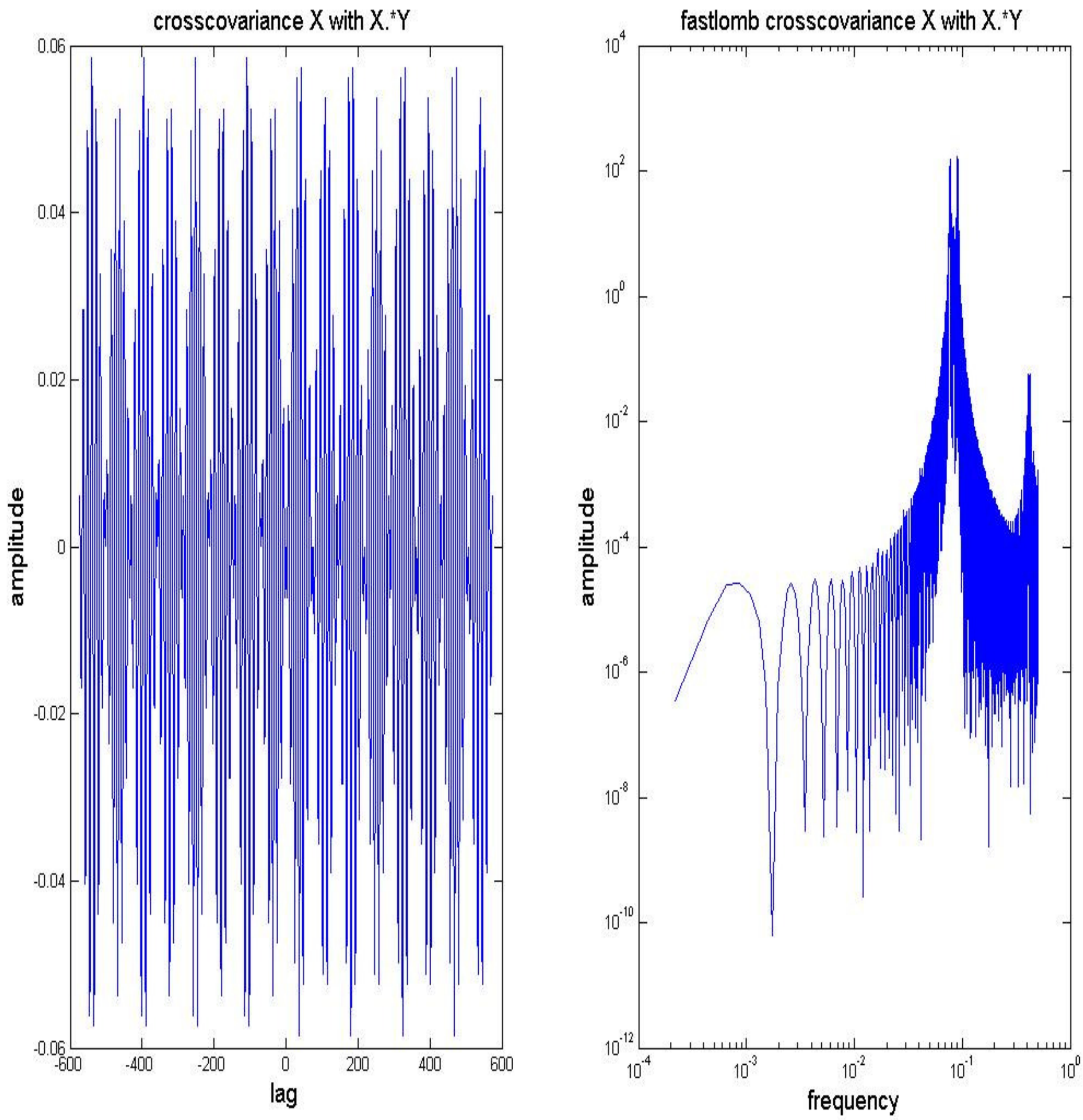


Figure 3-4

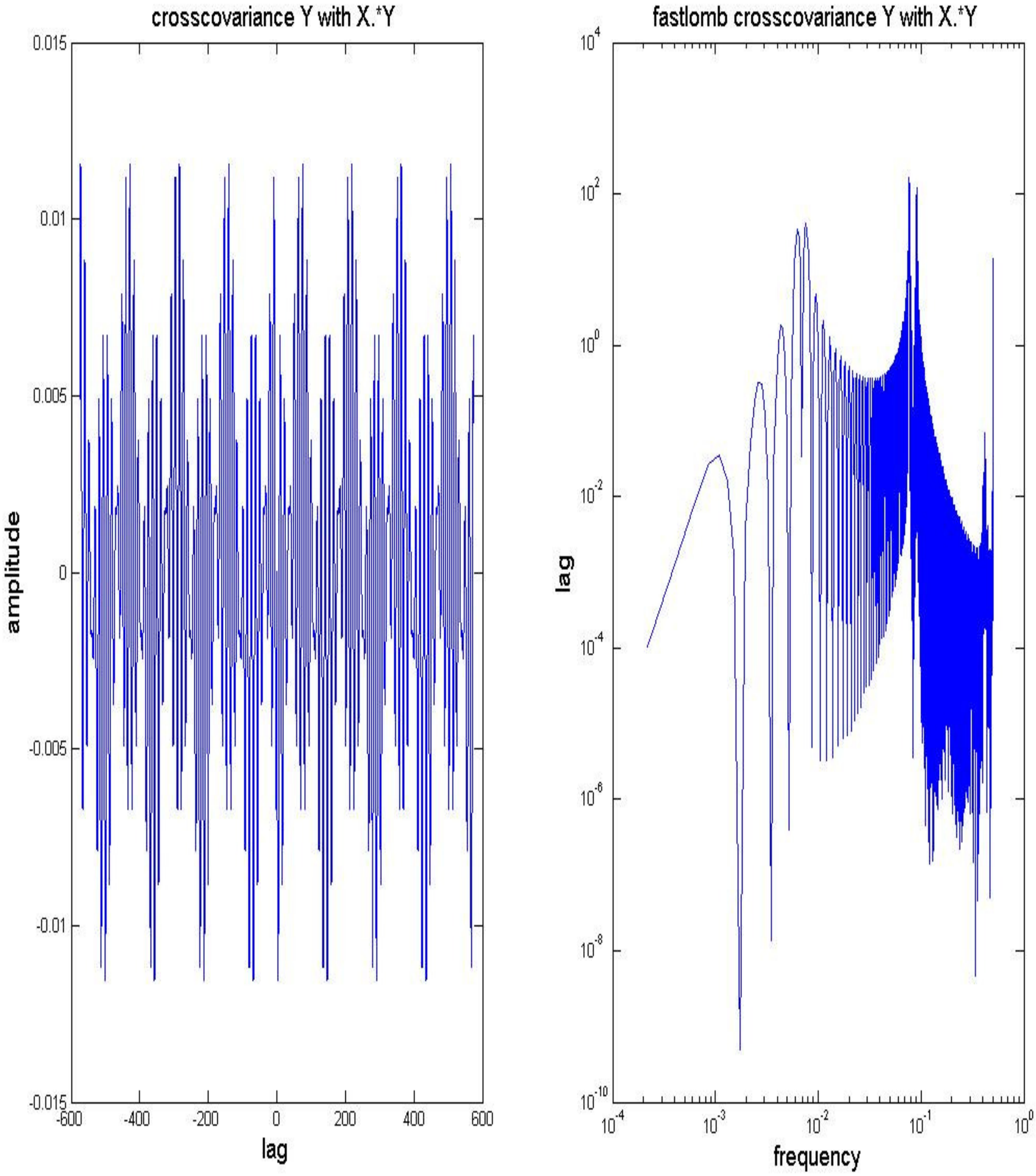

crosscovariance Y with X.*Y
amplitude
lag
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-600
-400
-200
0
200
400
600
fastlomb crosscovariance Y with X.*Y
lag
frequency

Figure 3-5

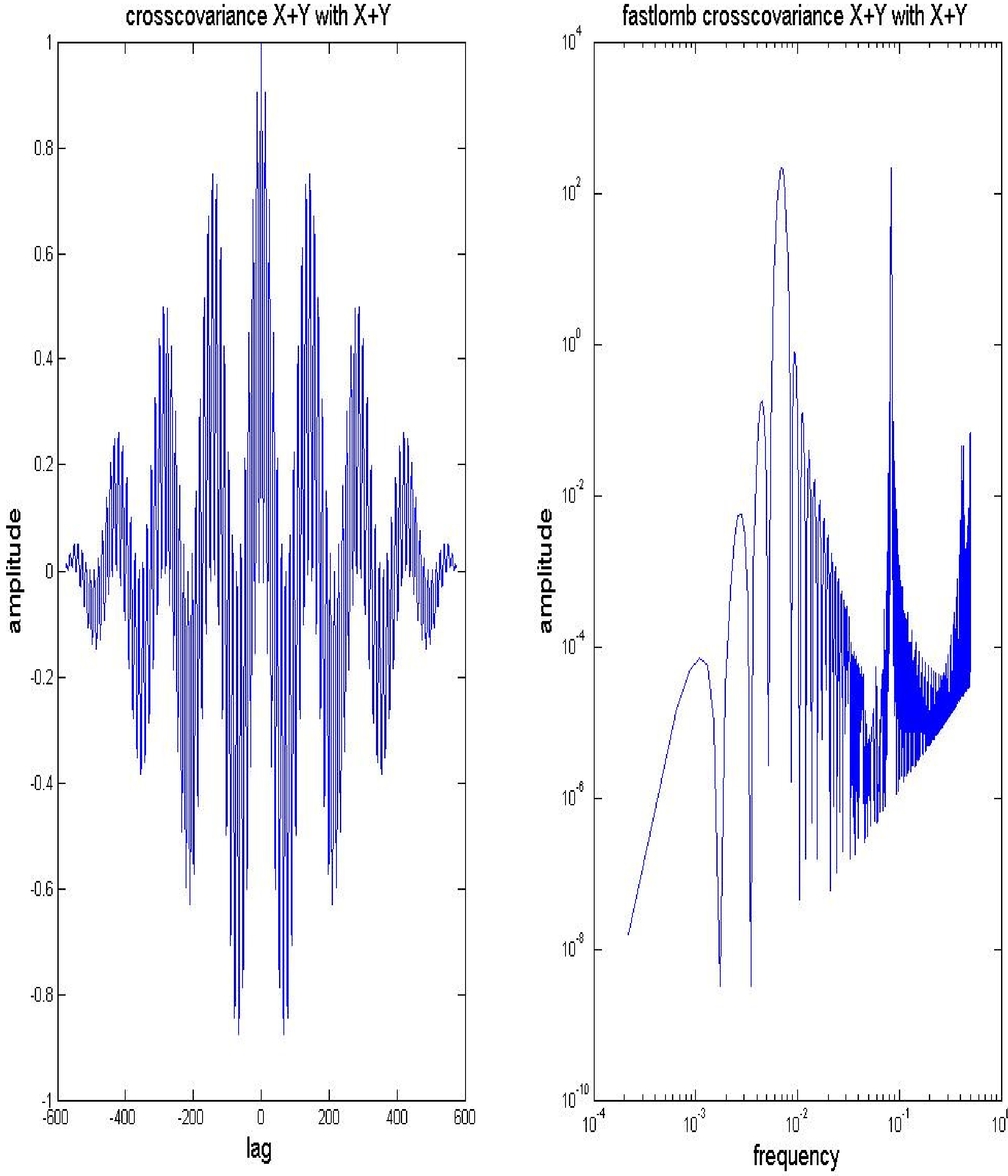

crosscovariance X+Y with X+Y
amplitude
lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-600
-400
-200
0
200
400
600
fastlomb crosscovariance X+Y with X+Y
amplitude
frequency
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

Figure 3-6

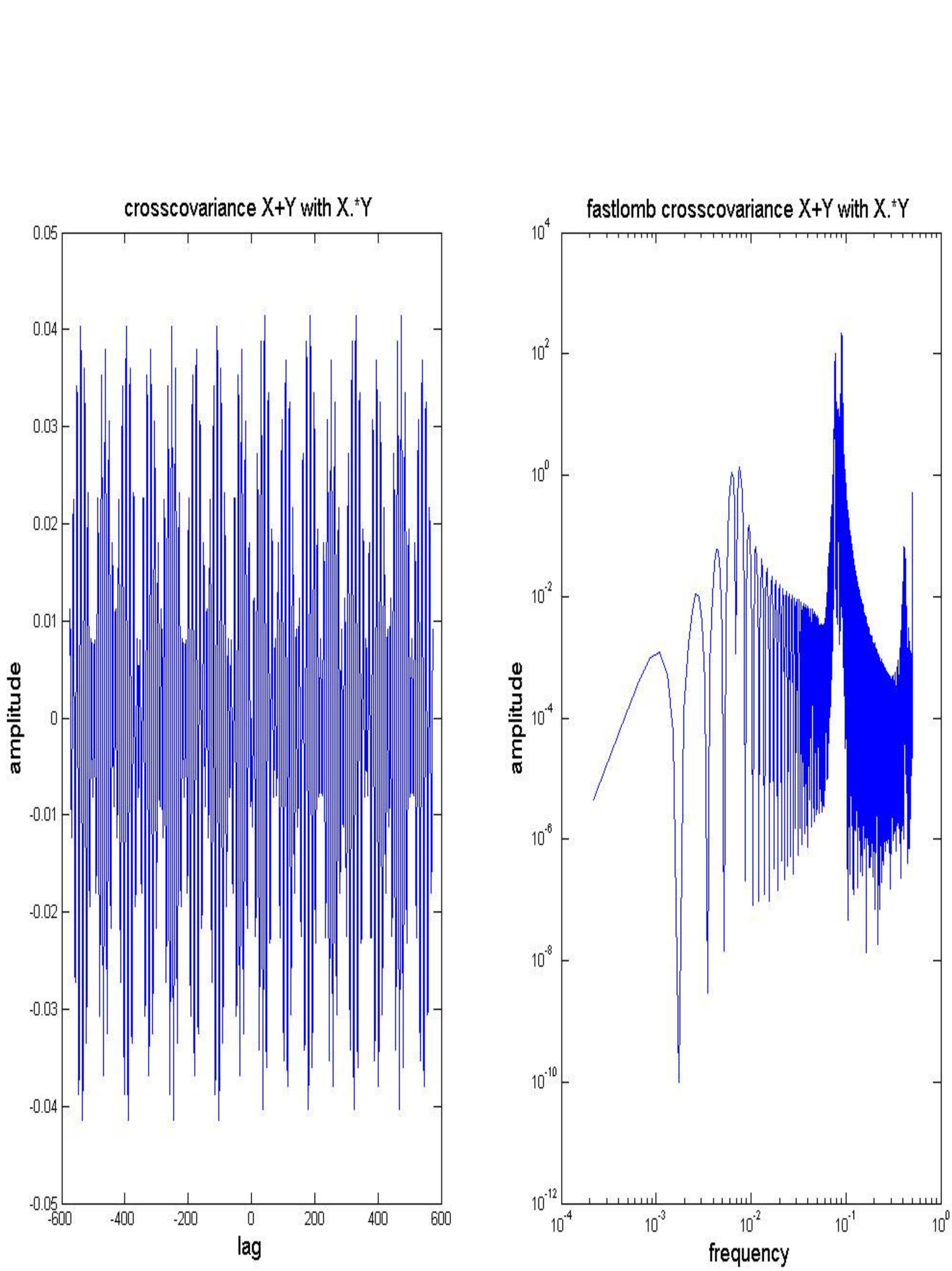

crosscovariance X+Y with X.*Y
amplitude
lag
fastlomb crosscovariance X+Y with X.*Y
amplitude
frequency

Figure 3-7

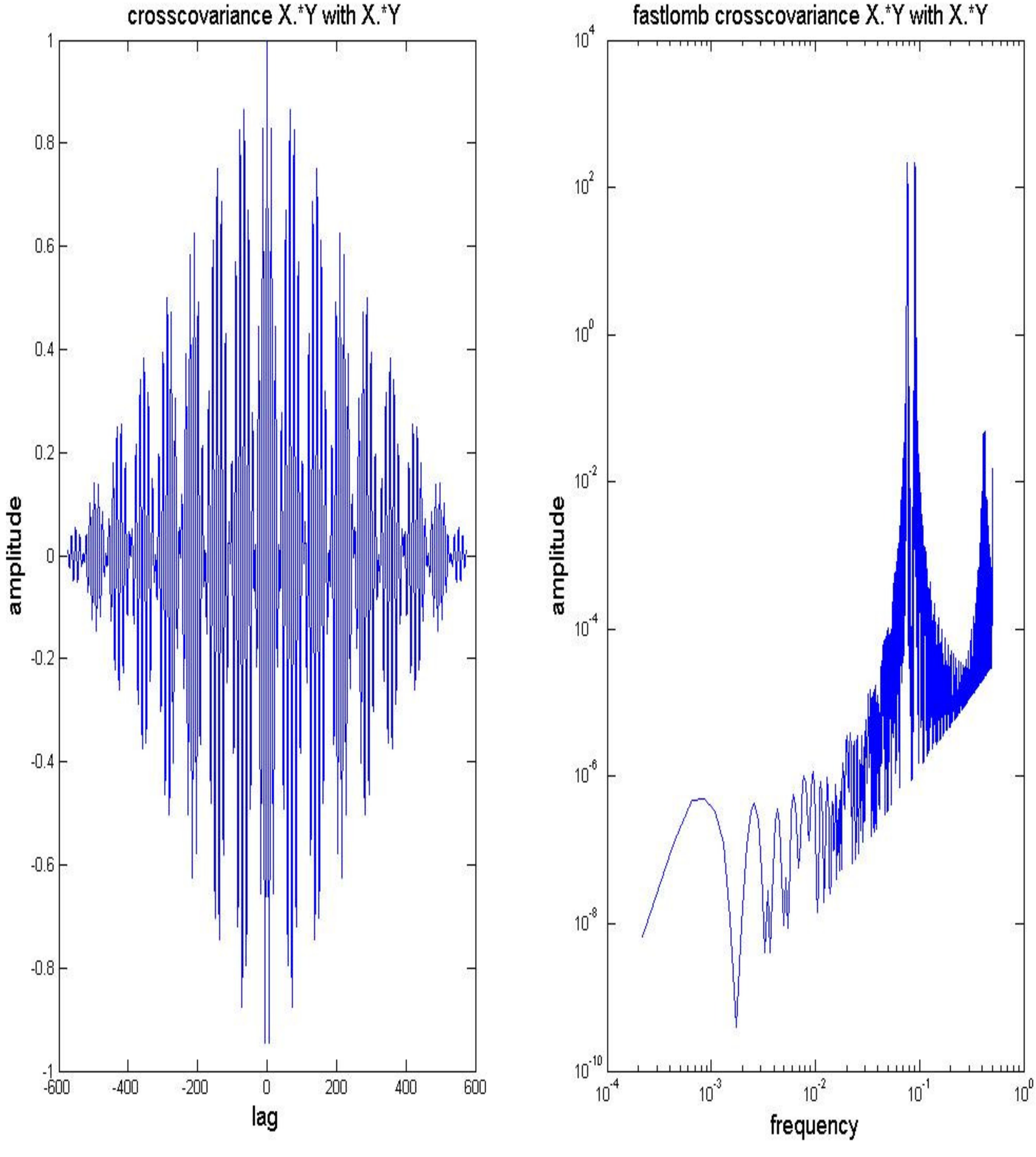

crosscovariance X.*Y with X.*Y
amplitude
lag
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-600
-400
-200
0
200
400
600
fastlomb crosscovariance X.*Y with X.*Y
amplitude
frequency
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

Figure 3-8

4. Data Evaluation and Results

4.1. Introduction

In this chapter the data evaluation and the results of this project will be given.

The ambition is to find out to what extent the QBO influences the Antarctic planetary waves in the outer to central area of the polar vortex (40°S to 60°S). This was accomplished by analyzing the zonal wind velocities, temperatures, geopotential heights and ozone concentrations in a 32 year section of the ERA40 dataset [The ERA40 re-analysis, Materials and Methods]. This was compared to the 32 years section of the QBO winds from the Free University Berlin [The QBO data, Materials and Methods].

The techniques used for the analysis were introduced in the section “Materials and Methods”, and the programs themselves are given in the appendix. Here their application will be given by dividing the analysis into two parts, the identification and extraction of the waves, and the analysis of the time series of the PW amplitudes to see the effects of the QBO on them.

The required ERA40 datasets for all variables (O_3–concentration, longitudinal wind velocity, geopotential height and temperature) over the entire 32 years are too big for downloading at one time (the download size on the webpage is restricted), the data for each variable were downloaded separately and, for reasons of clarity, treated in separate Matlab files.

4.2. Extracting the planetary waves from the mean flow

In order to examine the effect of the QBO on planetary waves, it was first necessary to identify which longitudinal (zonal) oscillations were true planetary waves according to the definitions given in the introduction. This section describes the Fourier analysis and amplitude and phase fitting that were performed on a subset of the data to extract the individual zonal oscillations. Those oscillations, which demonstrated consistency in altitude and westward propagation were then taken to be the planetary wave numbers to be extracted and analyzed over the entire 32 year dataset.

4.2.1. Importing the data into Matlab

For importing the datasets the netCDF reader was used to load the data for the entire globe into Matlab, but the ambition of this work is -as mentioned above- the polar outer to central part of the vortex between 40°S and 60°S figure (2-4). To enhance the signal to noise, the data taken at every 2.5^{o} of latitude were then averaged over this latitudinal section. The mean over longitude was subtracted off so that the longitudinal oscillations could be studied.

The July 2000 temperature data were used as an example year. This year was chosen as no special events, such as a polar stratospheric warming, occurred, and there was good data coverage for input to the ECMWF model, increasing its accuracy. July was chosen because it is mid-winter in Antarctica when planetary wave propagation is at a maximum [Pancheva 2004]. The resulting data as a function of longitude for each day of July were treated separately. This was done for each of the four different pressure levels, at 1, 10, 50 and 100 hPa, and each pressure level was analyzed separately. These data were then analyzed using two programs: the "fft_apodization_average_latitude_July_2000_t" and the"amplitude_32_years_t".

.

4.2.2. fft_apodization_average_latitude_July_2000_t for identifying the PW

In this program the Fast Fourier Transform (FFT) [The Matlab FFT function, Materials and Methods] was performed over longitude of the temperature data for each pressure level to identify the zonal wave numbers of the oscillations present in the data. Once the data were read in, they were windowed [The Matlab Apodization function, Materials and Methods] with a function that tapered to zero at the ends of the dataset. This was done to remove transients at the ends of the dataset that would create additional noise in the FFT [Orfanidis, 1995]. The data were then transformed, and the results are presented in figure (4-1).

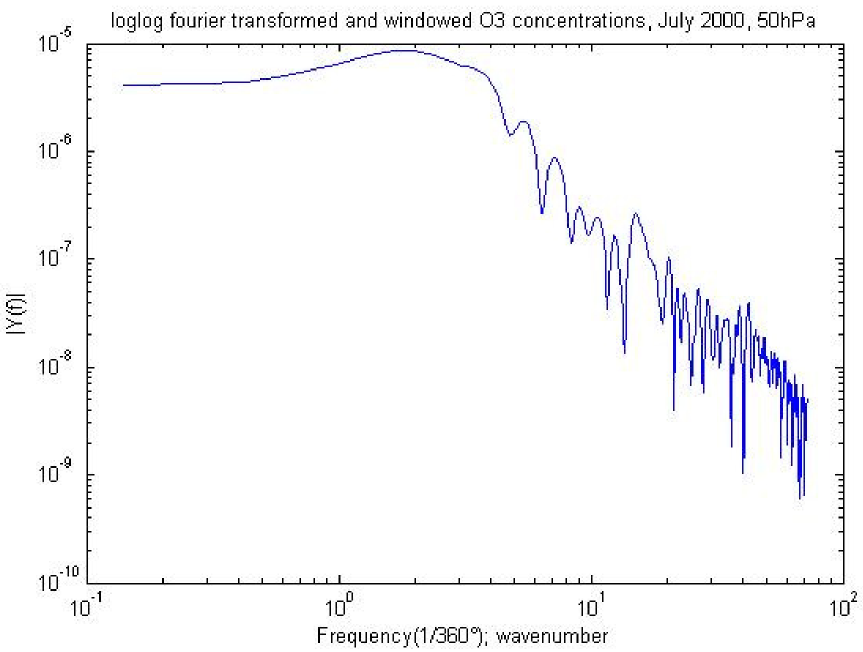


Figure 4-1 the windowed and fourier transformed O_3 concentration for July 2000

Here one can see in the double logarithmic plot that basically the amplitude decreases substantially after the wave number 4 (the x-axis is given in cycles per 360°, which is the same as the zonal wave number). For the data shown, the strongest wave number appears to be wave number 2, which is consistent with the longitudinal pattern of temperature at constant latitude shown as a function of longitude in figure (4-2) To be sure that all waves were accounted for in the further analyses, waves up to wave number 6 were treated.

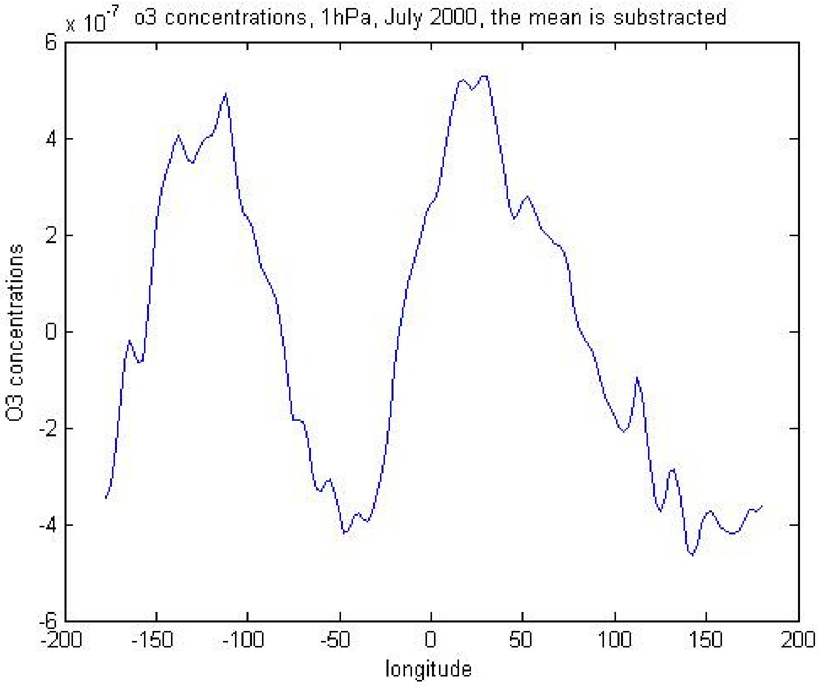


Figure 4-2 example of the ERA 40 data. The graph shows the O_3-data -averaged over days and latitude- over longitude from the level of 1hPa from the 21. July 2000

4.2.3. amplitude_32_years_t for extracting the PW amplitudes

The Quasi-Biennial Oscillation should influence the planetary waves, but what exactly does it influence?
The zonal wave number is constant since it needs to be an integer. Therefore the wave number will not be influenced. In addition, the waves are present most of the time in the winter, and therefore their occurrence frequency will not be amplified. However, the amplitude of the planetary waves can be influenced by the QBO through ducting, and thus make a difference in the polar vortex.
Therefore in this program the PW amplitudes and phases for wave numbers 1-6 were fit to the longitude data at each altitude, and the waves reconstructed for the different pressure levels and wave numbers. This allowed us to check if the detected oscillations were planetary waves. That is to see if the amplitude was persistent in altitude and if there was a westward propagation. The fitting of the amplitude and phase of the detected wave number oscillations was done by using the Levenberg-Marquardt fitting algorithm.

The Levenberg-Marquardt fitting algorithm was applied to the July data for each of the 32 years from 1971 to 2002 individually. To employ this fitting algorithm, a function that the algorithm would fit the data points to needed to be created from the equation:

$$f(w,x) = w_0 + w_1 + w_2 + w_3 + w_4 + w_5 + w_6 \quad (4\text{-}1)$$

with

$$w_0 = x(1)$$

$$w_1 = x(2) * \cos(\frac{2\pi * longitude}{360}) + x(3) * \sin(\frac{2\pi * longitude}{360})$$

$$w_2 = x(4) * \cos(\frac{2\pi * longitude}{180}) + x(5) * \sin(\frac{2\pi * longitude}{180})$$

$$w_3 = x(6) * \cos(\frac{2\pi * longitude}{120}) + x(7) * \sin(\frac{2\pi * longitude}{120})$$

$$w_4 = x(8) * \cos(\frac{2\pi * longitude}{90}) + x(9) * \sin(\frac{2\pi * longitude}{90})$$

$$w_5 = x(10) * \cos(\frac{2\pi * longitude}{72}) + x(11) * \sin(\frac{2\pi * longitude}{72})$$

$$w_6 = x(12) * \cos(\frac{2\pi * longitude}{60}) + x(13) * \sin(\frac{2\pi * longitude}{60})$$

Here the longitude was an array of a complete cycle with step of 2.5^o, with x ranging in index from (1,…,13) gave back the amplitudes for wave number 1 to 6.

There arose some problems with the fitting algorithm when large data values were fitted. In those cases the algorithm, a Matlab user library function, returned a too high sum of squares of equation residuals. This made it necessary to test the results for non-convergence of the fit, and to divide the data by powers of 10 until the individual values were smaller than 10^{-3} before fitting.

If the algorithm did not converge, the warning: ”The fitting algorithm didn’t work correctly, probably o3_d_altlongsst needs to be divided by a power of 10 (don’t forget to multiply amp and amp_phase by the same number again)” would have been given out to make it possible to fix these problems.

To determine if the oscillations present in a given year were westward propagating planetary waves, the reconstructed oscillation for each wave number was plotted as a function of longitude for each day of that year’s July on a contour plot. The peak amplitude of westward propagating waves would then progress toward the west as time increased, displaying a phase tilt to the left. This process was repeated for each altitude to ensure that the wave was present in the altitude region of the polar vortex. Only oscillations that were propagating westward and present at all altitudes were identified as planetary waves and analyzed further.

There arose the problem that when the fitted $A\cos(x)+B\sin(x)$ waves were reconstructed in the form $C\cos(x+\phi)$ where $\phi=-\tan^{-1}(A/B)$ is the phase angle and $C=\sqrt{A^2+B^2}$ the amplitude, there appeared phase jumps in the contour plots which should not have been there (figure (4-3)). These phase jumps were jumps of 180° and came from the form of the reconstruction, which never can give back negative amplitudes C since it is a square root of two real quadratic terms.

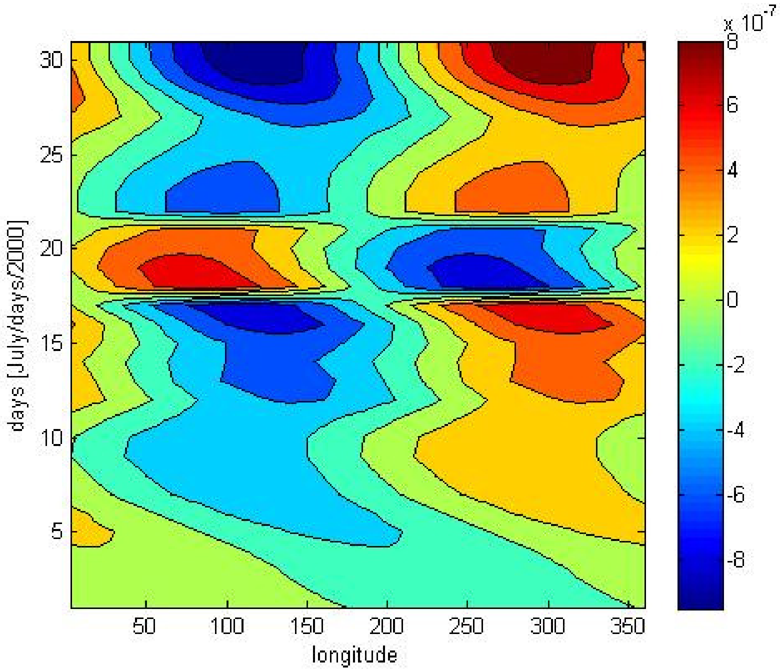


Figure 4-1 example for the phase shift under the reconstruction of PW data separated in wavelength fitted to the form A*cos(x)+B*sin(x), reconstructed in the form C*cos(x + ϕ)

Therefore the form $C=\sqrt{A^2+B^2}$ was used in further analysis only where changes in the size of the amplitude were important and not the phase. However for the wave propagation and altitude consistency on the contour plots, the form $A\cos(x)+B\sin(x)$ was used, since this avoided phase jumps in the plots.

The results of this analysis can be seen in the figures (4-4) and (4-5). Figure (4-4) shows the results of the Levenberg-Marquardt fitting algorithm where the blue line shows the original data and the red line the curve that was fitted to it. As one can see the main harmonics are copied by the LMF algorithm and the small amplitude sub harmonics, including some noise, were filtered out since these probably are no significant planetary waves.

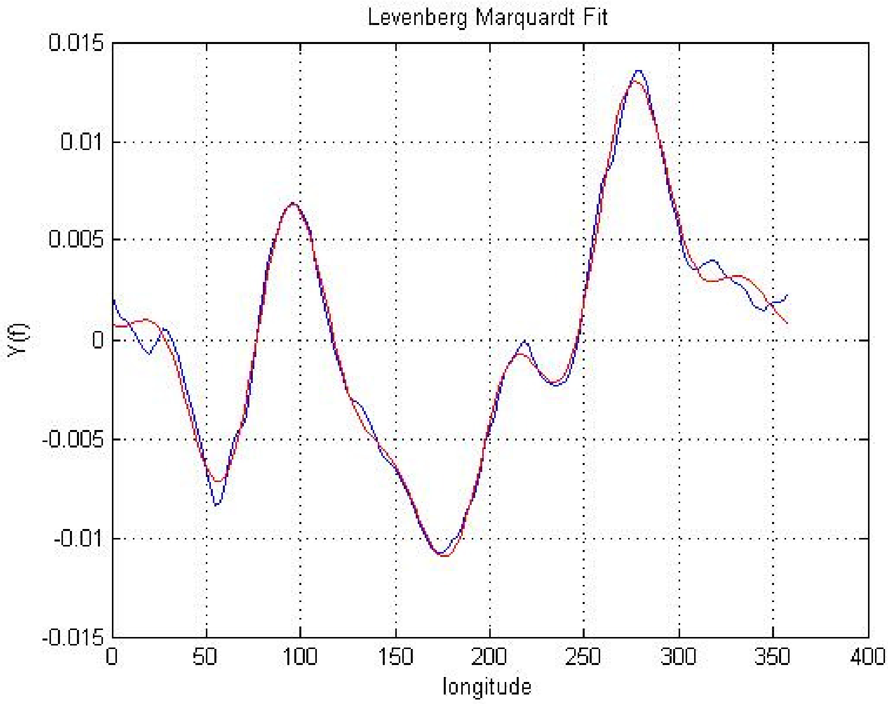


Figure 4-2 example for a Levenberg-Marquardt fit of the O_3-concentration over longitude. The blue line gives back the original data and the red line the fitted data

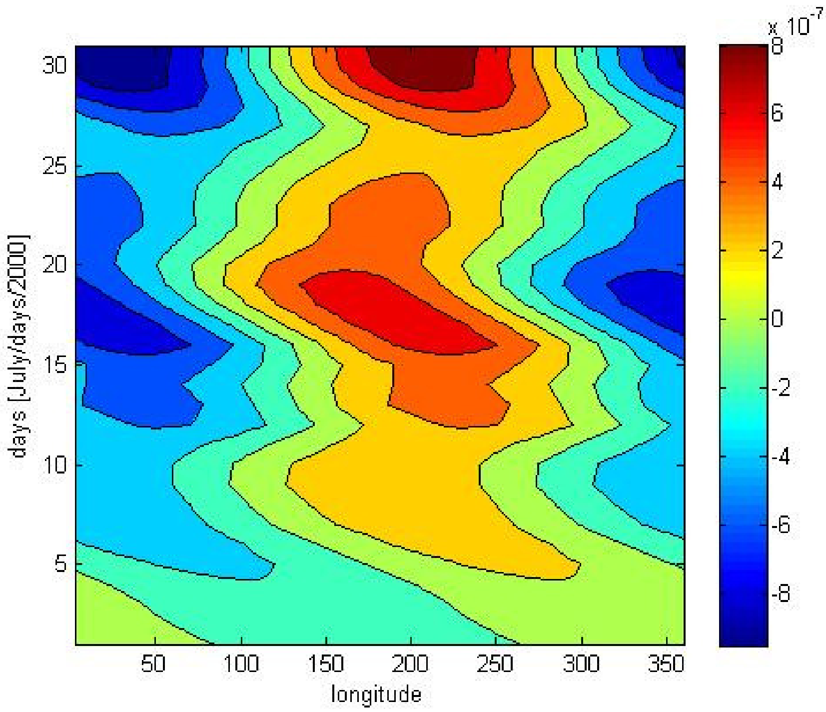


Figure 4-3 example for the reconstruction of PW data separated in wavelength fitted to the form A*cos(x)+B*sin(x), reconstructed in the same form

Figure (4-5) shows a working reconstruction of the planetary wave plotted over longitude and time. The phase shift appearing in figure (4-3) is corrected by using the reconstruction of the form $A\cos(x) + B\sin(x)$.

Up to this point we have been dealing with daily data for each July. However, in order to compare the variations in the PW amplitudes with the wind changes associated with the QBO, it was necessary to form a time series of the PW amplitudes over the 32 years of data. The wave amplitudes for the wave numbers identified as planetary waves above were summed for each day of July and then averaged over the entire month. This created the average planetary wave amplitude for July of each year. The July data were used in the subsequent time-series analysis as the waves are strongest and most persistent during mid winter. Finally the variables needed for further analysis were exported into a mat-file from which they could be imported without re-doing the time-consuming amplitude extraction (approximately one hour) that has been described here.

As a first step, a FFT of the PW amplitude data could be performed to examine the periodic temporal variations present. These could then be compared with those present in the Quasi-Biennial Oscillation. The program fft_apodization_average_latitude_July_2000_t could also be used to form the FFT of the time based series, where the transformed results were in frequency (or its inverse, period). The results of this transform of the July data are shown in figure (4-6). This shows a strong and significant peak at about 2.4 years, the period of the Quasi-Biennial Oscillation.

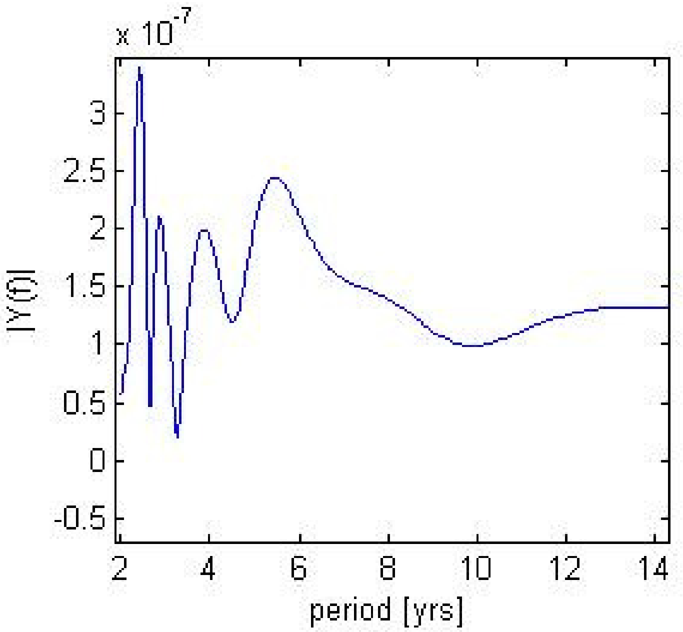


Figure 4-4 detail of the FFT of the PW amplitudes on the level 50hPa and of the wave number 3. One can clearly recognice a strong peak at about 2.4 years

4.3. Analyzing the PW amplitudes

The significant peak at the period of the QBO found in the time series of the PW amplitude data, while promising, still is a weak argument for a relationship between the two. In addition, it does not tell anything about how the QBO may influence the PW. Therefore in this section two programs will be described. In those programs was performed a cross spectral analysis in order to determine whether the oscillations in the two datasets were phase coherent, an indication that they are indeed related. Further, it was checked if an averaging of the amplitudes over wave numbers and pressure level would decrease the noise in the cross-spectral analysis, increasing the significance of any relation between the amplitudes and the QBO.

4.3.1. QBO for finding out about the coupling between the PW and the QBO

In this program was made the cross spectral analysis i.e. a cross covariance between the PW amplitudes and the QBO and adjacent a FFT of that.

Once the planetary wave data for every variable (temperature, geopotential height, wind velocity, ozone concentration) were read in from the before saved mat file, the 19 pressure level QBO data were imported. They were sampled monthly and therefore needed to be shortened to annual dependency. Therefore the July data (same month as the PW data were sampled to) were extracted out of the entire time scale. It was made a cross covariance for each of the 19 levels of the QBO. That was done for each of the 4 pressure levels at each of the 6 wave numbers. Then, to find out if the PW and the QBO were related, an FFT was performed on this cross covariance to get the cross spectra in time.

The results of the cross covariance and the FFT cross-spectra were plotted into the figures (4-7) and (4-8). One can see a period in the contour plot that comes out to be about 2.4 years. But as well one can see a second periodicity between 10 to 15 years. That second frequency varies the maximum amplitude of the 2.4-year oscillation.

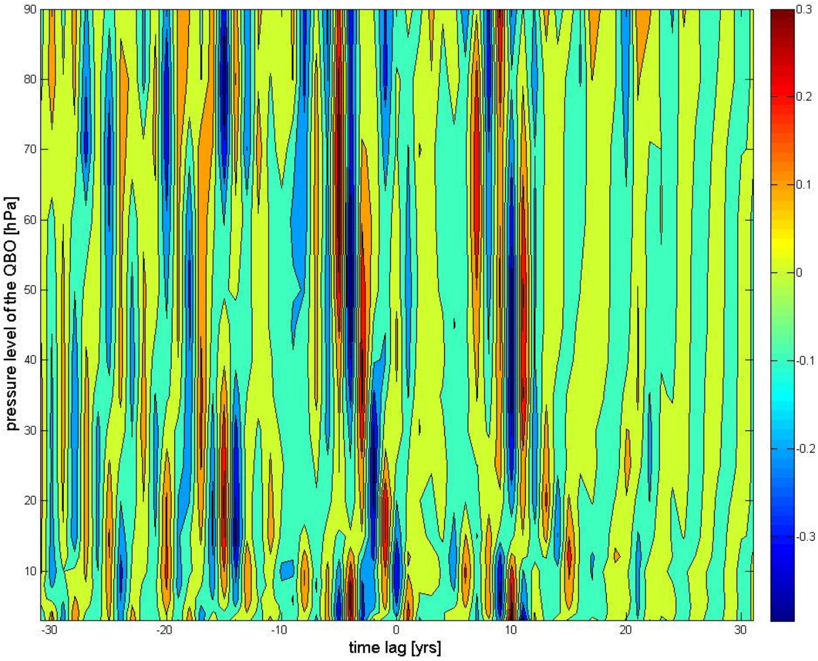


Figure 4-5 xcov between the QBO and the 10hPa PW amplitude of wavenumber 2. The few more than 2 years periode of the QBO is good vivible but as well another period with about 10 to 15 years (between the red parts)

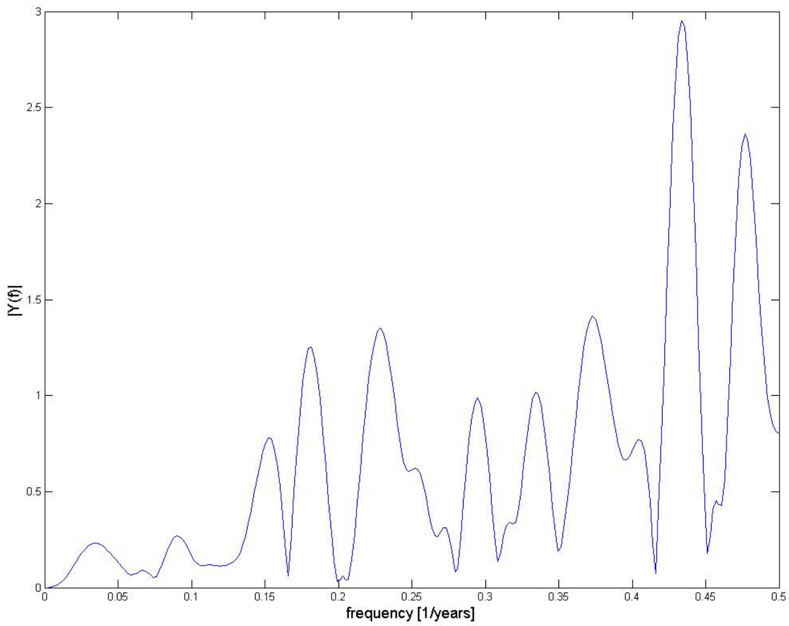


Figure 4-6 FFT of the xcov between the QBO at 40hPa and the PW of wave number 6 at 100hPa

The FFT plot over the same time does show two strong peaks centered at a frequency of about 0.44 yrs^{-1} (2.22 years period) which is about where the 2.3 year periodic Quasi-Biennial Oscillation should show up. However, there is no significant peak at 0.1 yrs^{-1} (10 years period), where the peak of longer time scale oscillation should show up. One possible reason for that is explained in 3.2.5. Thereafter it is possible that the in the time domain visible 10 to 15 years frequency is a modulation frequency due to the sampling period of 1 year.

Since the same behavior is observed at all levels, the planetary wave amplitudes were averaged over wave numbers and pressure level and a cross covariance between that and the QBO was done for each pressure level of the QBO. The purpose of that is to see in the next program if that can cancels out noise. Therefore the needed variables were exported into a mat-file.

4.3.2. phase_between_QBO_and_PW

In this program was tried to answer two questions: How big the degree of covariance between the averaged planetary wave amplitudes and the Quasi-Biennial Oscillation winds is at zero lag, and at which altitude the degree of covariance or anti-covariance reaches its maximum (it was looked for an anti-correlation to prove the hypothesis that westward (negative) QBO phase causes maximum south polar planetary wave activity) .

Thereby was first tried to average the planetary wave amplitudes over all pressure levels and wave numbers and answer the questions at a final covariance contour plot.

As one can see in figure (4-9-a and 4-9-b) does indeed an averaging over the pressure levels and wave numbers of the planetary waves cancel out noise since an auto covariance of the QBO has mostly the same peaks as a cross covariance of the QBO and the planetary waves.

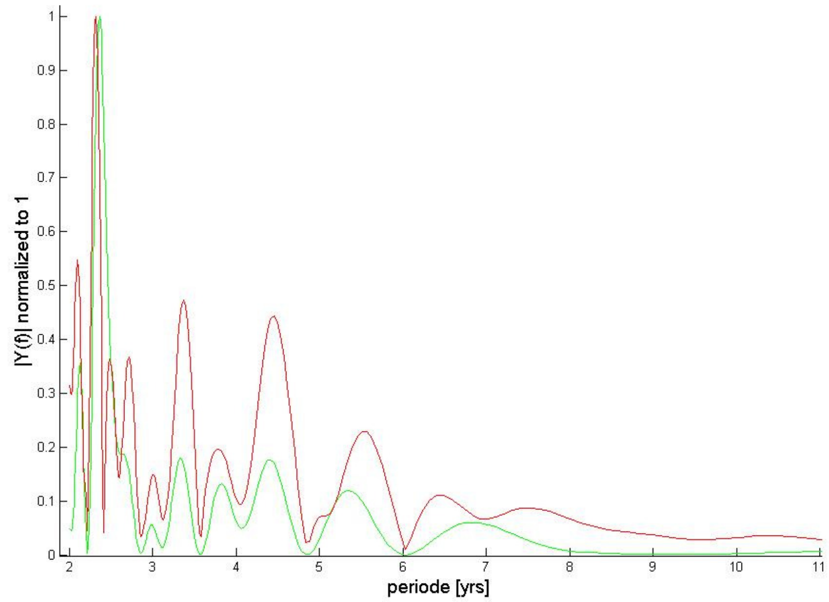


Figure 4-7-a In red is shown the cross spectrum between the planetary wave amplitudes, averaged over wave number and pressure level, and the QBO. Also shown in green is the FFT of the QBO

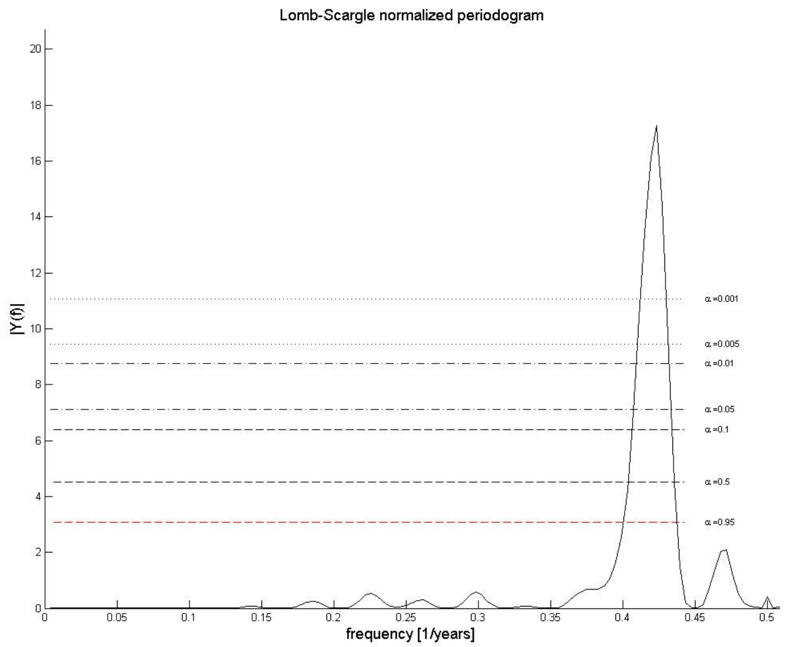


Figure 4-9-b lomb periodigram of the QBO for the pressure level 45hPa. One can clearly recognize the strong peak at 2.359 years

But there came up two problems:

1. The modulation frequency occurring as a splitting of the peaks in a cross spectra can affect the sharpness of the used contour plot.
2. The upcoming beating frequency discussed in chapter 3 can as well affect that sharpness since we want to do an analysis in the time domain.

The solution for the first problem was to choose the -with equation (3-5) calculated- center peak at f_0 (0.46 *yrs* $^{-1}$) in the cross spectra between 0.40 *yrs* $^{-1}$ and 0.37 *yrs* $^{-1}$ and to interpolate the cross covariance between the Quasi-Biennial Oscillation and the averaged planetary waves. Therefore was used the Levenberg-Marquardt algorithm. Only one single peak was required since more peaks would take into account the splitting like in figure (4-10) shown for 3 peaks at 2.70 years, 2.50 years and 2.17 years. Those were about the periods of the 3 most significant peaks for the cross covariance between the averaged planetary wave amplitudes and the QBO at all levels of the QBO.

Problem 2 was solved by using a higher sampling frequency for the reconstruction of the one single wave. Figure (4-11) shows the result of that.

Now one can read off that the correlation between the QBO and the averaged planetary waves is mostly negative and the strongest negative correlation occurs at 50hPa and stays nearly constant until higher altitudes of about 40hPa.

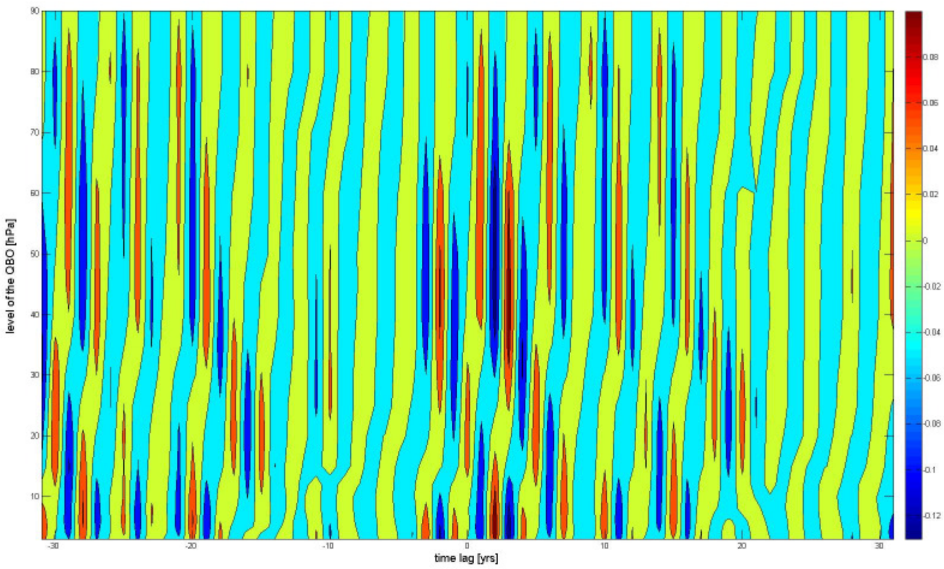


Figure 4-8 contour plot of the 3 waves fit from the xcov between the averaged PW amplitudes for the variable t and the QBO

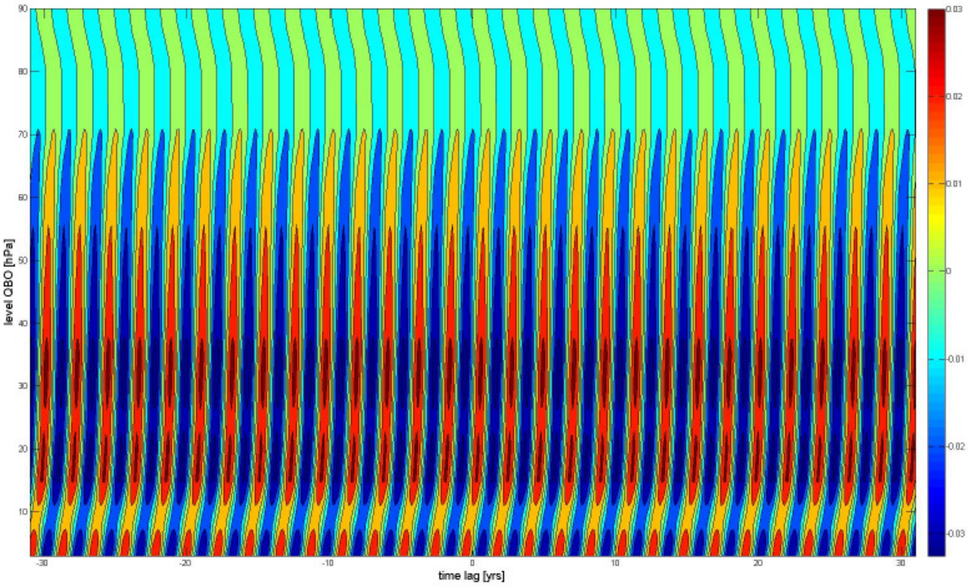


Figure 4-9 contour plot of the one wave fit from the xcov between the averaged PW amplitudes for the variable t and the QBO

5. Discussion

5.1. Introduction

The discussion will be split into three parts: the Identification and the two parts of Analysis. In the Identification will be discussed why which wave orders were taken into account for analyzing planetary waves. In the Analysis will be discussed if there is a visible influence of the Quasi-Biennial Oscillation on the Antarctic planetary waves, if there are upcoming longer time scale periods and where the influence does take place.

5.2. Identifying the planetary waves

Figure (4-1) shows the spreading of the amplitudes for all zonal wave numbers that were found in the polar vortex. It is easy to see in figure (4-1) and (4-2) that wave number two is strongest. Wave numbers 3 and 4 are strong as well and wave number 5 and 6 are very weak but still taken into account since 2,3,4,5 and 6 show all very strong westward propagation what was the criteria for being a PW during the discussion of planetary waves (exemplary is wave number two given in figure (5-2)).

The only arguable point is wave number 1 which show a very weak westward propagation as in figure (5-1) but is still taken into account since its amplitude is much smaller in comparison to the strongest wave number 2. Therefore was concluded that the extracted wave numbers 1, 2, 3, 4, 5 and 6 are identified as planetary waves and used for the further analysis.

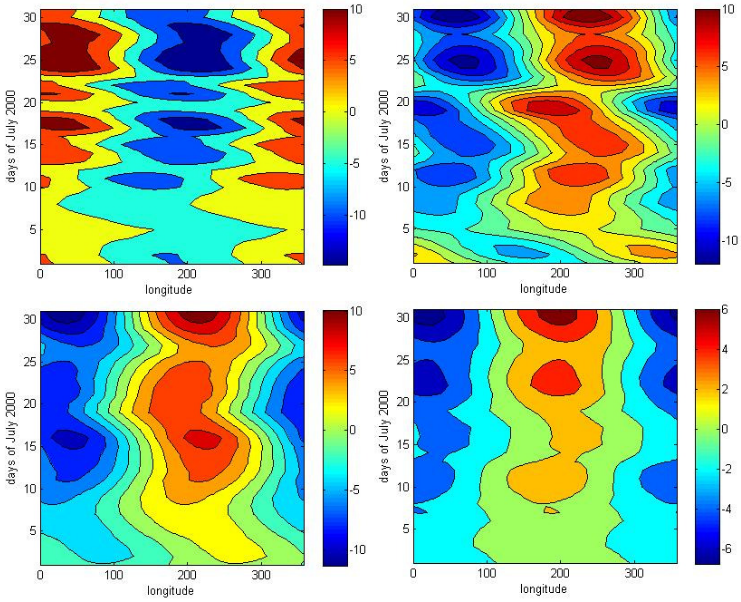


Figure 5-1 Hovmöller contour plots of wave number 1, altitudes are from the top left to the bottom right 1, 10, 50, 100 hPa for the temperature in July 2000

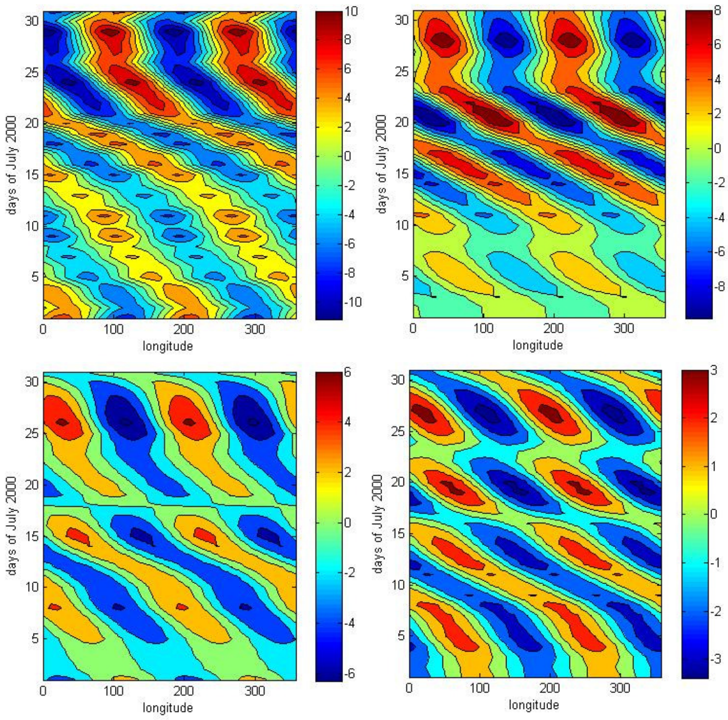


Figure 5-2 Hovmöller contour plots of wave number 2, altitudes are from the top left to the bottom right 1, 10, 50, 100 hPa for the temperature in July 2000

5.3. <u>Analyzing the planetary waves</u>

First was tried to identify the Quasi-Biennial Oscillation in the planetary wave by using FFT on the averaged planetary-wave amplitude time series. This was done to check if there is visible a strong peak on a 2 years period, for all wave numbers and all pressure levels. Figure (4-6) shows an example of that. As one can see appears the most significant peak at about a period of 2.3 years. As mentioned in chapter (4.3), the existence of this peak is, while promising, is not strong enough evidence that the Quasi-Biennial Oscillation influences the planetary waves as proposed in chapter 2.6. Further evidence is given by the period of about 2.3 years found in the cross-covariance as a function of lag shown in figure (4.7). But one can also recognize a second oscillation with a period nearly 10-15 years. Speculations about the origin of these two oscillations lead of course to the Quasi Biennial Oscillation and another strong geophysical event that has a period between 10 and 15 years: The Solar Cycle. In order to determine if there is indeed a phase coherent oscillation between the QBO and the planetary wave amplitudes, and to determine if there is a modulation due to the solar cycle, cross spectral analyses were computed.

From a Fourier analysis of the Quasi-Biennial Oscillation for each level the most significant peak is often located at 2.4 years or 0.4239 years^{-1} (compare to figure (4-9-b)).

And indeed the strongest peak of the Fourier transform of the cross covariance between the QBO and the planetary wave amplitudes (the cross spectrum) is also at the frequency as one can see in the figure (4-8). This would indicate (using the tests done in appendix 3) that there is a direct, phase coherent modulation of the QBO and the planetary waves. However, a peak at 11 years (or a frequency of 0.091 yrs^{-1}), where the Solar Cycle is supposed to be, does not show up. This disappearance of the lower frequency peak can be used to determine how the coupling between the long and short frequency waves shown in the co-variance vs. lag (figure (4-7)) comes about.

The only two couplings where the lower frequency in a cross spectrum peak totally disappears are the couplings $X*Y \otimes X*Y$ $(coupling1)$ in figure (3-8) and $X \otimes X*Y$ $(coupling2)$ in figure (3-4). There the higher frequency peak always splits in two peaks that are seperated by

$$\frac{1}{f_m} = \frac{2}{f_u - f_l} \qquad \text{(5-1)},$$

where f_m is the modulation frequency (the lower frequency and f_u respectively f_l are the two peaks that result from splitting of the higher frequency peak). When one looks at the cross covariance plot one can clearly exclude the coupling 2 mechanism. It was shown in section (3.2.5.) that *coupling 2* results in values for the maximum cross covariance that are smaller than 0.05, even when no noise is existent, whereas the values here, shown in figure (4.7), are over an order of magnitude larger. Therefore, the only possible coupling that would explain the disappearance of the low frequency peak and the splitting of the higher frequency peak is the purely multiplicative *coupling 1*.

Given that multiplicative coupling we can calculate where the two peaks in the cross-spectra should be found if the solar cycle is modulating the coupling between the QBO and the planetary-wave amplitudes. From a Fourier analysis of the F10.7 dataset over the last 52 years, a very significant period for the solar cycles was found at 10.0281 years (figure (5-3)). Putting that into equation (5-1) with the frequency of the QBO peak at 0.4239 yrs^{-1}, the QBO should be split into two peaks spaced equally in both directions by 0.0997 years^{-1}.

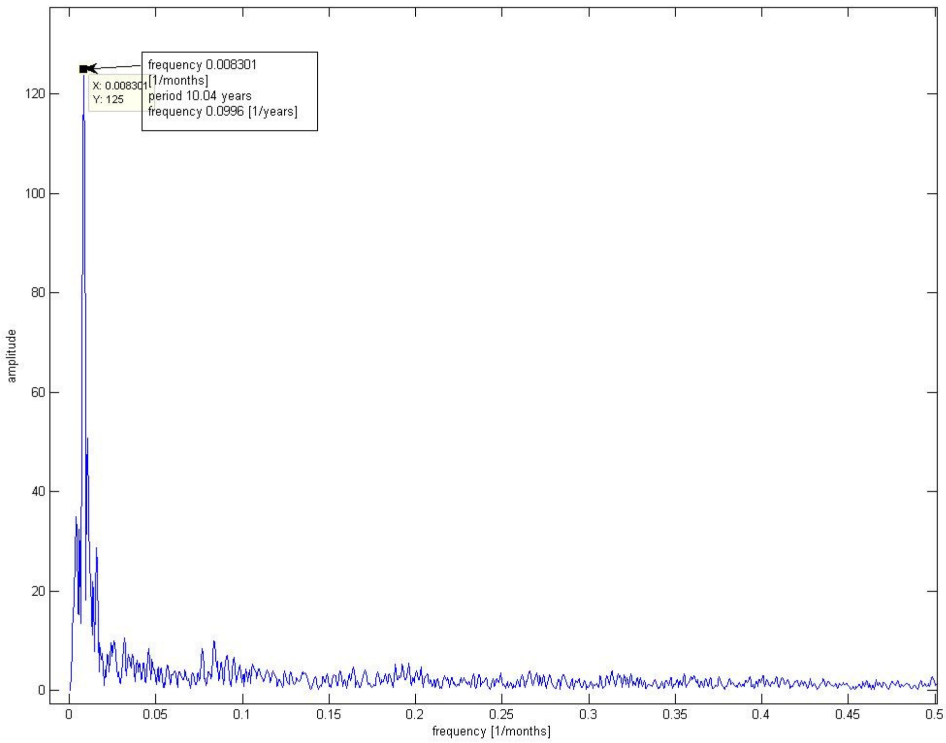


Figure 5-3 frequency spectrum of the F10.7 solar cycle documentation. One can clearly recognize the strongest peak at 0.0996 [yrs $^{-1}$]

Thus f_l should be 0.3242 years $^{-1}$ and f_u should be 0.5236 yrs $^{-1}$. However, f_u cannot be seen in the Fourier Spectrum any more since its frequency is higher than the Nyquist frequency. But since the Matlab fft function always computes two harmonics, the peak from the second harmonic should be visible in the first harmonic. As a picture, one can imagine, that the peak that actually lies outside of the spectrum becomes folded around the Nyquist frequency and appears at $2 * f_n - f_u$. Therefore f_u should show at 0.4764 yrs $^{-1}$. Indeed both split peaks do exist in the spectrum, but neither peak is significant in the spectrum, and they are much smaller than other peaks in the vicinity. Indeed, in chapter 3 it was shown that the 1 year sampling of the 2.3 year QBO will lead to the appearance of a modulation in the cross-covariance, but will not generate a spectral peak in the Fourier transform of the cross-covariance. Thus, while there might be a weak solar-cycle modulation of the coupling between the QBO on the planetary-wave amplitudes, it does not appear to be significant and its strong appearance in the cross-covariance is most likely an artifact of the sampling.

Another much more significant peak shows up in many cross spectra. It is, in the example spectra of figure (4-8), the second most significant peak. Its frequency is 0.4785 years $^{-1}$. Assuming it is one part of a split QBO peak again one would expect another peak at 0.3692 yrs $^{-1}$. And in fact there is a strong peak at 0.373 yrs $^{-1}$. Calculating the percentage error of the actual position of the peak and its theoretical position it is only 7.01%. Calculating the modulation period with equation (5-1) we find that the modulation period, T_m is at 18.32 years. Other workers have found this, as well as other long time scale periods, in models and identified them as Quasi-decadal oscillations resulting from equatorial winds changing with the period 18 years [Mayr, 2003]. This may be the cause of its appearance here, although it does not appear to be a strong feature in the QBO winds themselves, only in the cross covariance between the QBO and the planetary wave amplitudes. Other people have seen lunar effects like the lunar nutation on the climate variability. This effect has a period of about 18.6 years [Treloar, 2002] [Schumacher, 1999]. However, the mechanism by which a quasi-decadal oscillation or lunar periods would affect the QBO wind are not clear and, while noted here, their explanation is beyond the scope of this thesis.

5.4. Where does the QBO influence on the planetary waves take place in the atmosphere?

Earlier was clearly shown that the Quasi-Biennial Oscillation has a strong direct effect on the planetary waves, now can be tried to explain the consequences of the questions and answers found in chapter 4.3.2. Figure (4-11) shows the fitting of the Quasi Biennial Oscillation peak to the cross covariance between the averaged PW amplitudes and the QBO amplitudes. The fact that there is a phase-coherent oscillation in the QBO and the planetary wave amplitude demonstrates that they are certainly oscillating together, which itself is evidence that they are related. However, if the QBO is indeed modulating the planetary waves by ducting them to the polar regions during its westward, blocking phase, then there should be a definite sign of the cross-covariance at zero lag. Since the Westward phase of the QBO is defined as a negative wind, and a maximum of planetary wave amplitude in the polar regions would be a positive excursion from the mean, one would expect the cross-covariance to be both significant and negative at zero lag.

Indeed, the analysis in chapter 2.6.2, which used this sign convention, showed the zero phase shift (zero lag) value of the cross-covariance to be strongly negative. Hence, the planetary wave amplitudes and the QBO winds are anti-correlated. Therefore the hypothesis 2.9 “The Quasi Biennial Oscillation should block the Planetary Waves from propagating equatorward, in the case when the phase of the QBO is westward. This blocking and reflecting under westward QBO phase should show up in the data as a maximum in the Planetary Wave activity.”, is demonstrated.

The maximal negative value appears at a height between 40 and 50 hPa altitude of the QBO. Therefore this region is identified as the sign of the QBO defining altitude, what is consistent to the mostly used definition for that.

6. Summary and out view

The Quasi Biennial Oscillation does appear to have a strong influence on the amplitude of the Planetary Waves. The PW amplitudes almost perfectly copy the Quasi Biennial Oscillation like shown in figure (4-9).The strongest influence is from the 40hPa pressure level of the QBO. The additional peaks that appear in the cross-spectral analysis indicate that the influence of the QBO on planetary waves is modulated by other factors like the Solar Cycle and the Lunar Nodal. While these modulations may influence both the QBO and the planetary waves, and hence affect climate, the influence is very weak, especially for the Solar Cycle. Also, the appearance of the strong peak in the power spectrum at the frequency of the QBO indicates that the direct coupling of the QBO and planetary waves is much stronger than any multiplicative modulation.

Future analysis should be done on mechanisms by which the Solar Cycle and the Lunar Nodal could influence either the Quasi Biennial Oscillation, the planetary-wave amplitudes in the polar regions, or their coupling. This may lead to explanations of unusual events such as the South Polar Stratospheric Warming that took place in 2002.

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8. Appendix: Matlab Files

Here are given the for this work written and used Matlab files. If files were produced for every variable, the ones for the temperature are given exemplary.

8.1. fft_apodization_average_over_latitude_July_2000_t

Making a fft of the PW Amplitudes

```
%In this file is a fast fourier transform of the PW amplitudes doone in
%order to find out which frequencies do exist in the data (hopefully at
%least the abput 1/(2.4 years) of the qbo)
%To make the qbo look a bit clearer the data first have to be windowed,
%what means, that they have to be multiplied with a curve, that makes them
%become zero at the edges to avoid to much noise in the fft
```

loading in the ERA40 data set and creating links to the needed functions

```
addpath 'C:\Dokumente und Einstellungen\Tobias\Eigene Dateien\Studium\Bachelorthesis\Files_Jul_2000'

%Since the polar vortex is strongest at 60°S and it´s needed to find
%variations in the polar vortex, it will be most usefull to look at the
%edges



day=1;
max_lat=-40;
min_lat=-60;

%Extracting the data and puttig it into matlab variables
S=netcdf('Data_July');
[longitude,latitude,level,time,t,u,o3,z]=S.VarArray.Data;
```

```
% now get the attributes
att_longitude   = S.VarArray(1,1).AttArray.Val;
att_latitude    = S.VarArray(1,2).AttArray.Val;
att_level       = S.VarArray(1,3).AttArray.Val;
att_time        = S.VarArray(1,4).AttArray.Val;

% Temperature (K)
[t_scale, t_offset, t_fill, t_missing, t_units, t_longname]       = S.VarArray(1,5).AttArray.Val;

% U velocity (m s**-1)
[u_scale, u_offset, u_fill, u_missing, u_units, u_longname]       = S.VarArray(1,6).AttArray.Val;

% Ozone mass mixing ratio (kg kg**-1  )
[o3_scale, o3_offset, o3_fill, o3_missing, o3_units, o3_longname]  = S.VarArray(1,7).AttArray.Val;

% Geopotential (m**2 s**-2 )
[z_scale, z_offset, z_fill, z_missing, z_units, z_longname]       = S.VarArray(1,8).AttArray.Val;

% new format of net cdf files does not have level in pressure order, but
% instead in a sorted array, so, 1, 10, 100, 1000, 150, 2, 20 ...
% newlevel will be in sorted order, and IXX is the pointer into the array
[newlevel,IX]=sort(level,1);
```

Preparing the ERA40 data (shortening, averaging over latitude,...)

```
%loop for averaging the data over the latitude between the min and max
%value.

%loop over time
for d=1:4:size(time),
   %loop over scale height
for k=1:1:size(level,1),
```

```
    %loop over longitudes (stepwith 2.5°)
     for i=1:1:size(longitude,1),
         %needed temporary variables
           oo=0;
           ooo = 0;
           ninavg = 0.0;
           %averaging loop over latitude between min and max
           for j=1:1:size(latitude,1),
               %make shure we only averrage between min and max latitude
               if((latitude(j) >= min_lat) && (latitude(j)<=max_lat))
                   %values with a period of 6 hours are not needed and
                   %influenced by the sun, what leads to failures,
                   %therefore averaging over 1 day period

oo=(double(t(d,IX(k),j,i))+double(t(d+1,IX(k),j,i))+double(t(d+2,IX(k),j,i))+double(t(d+3,IX(k),j,i)))/4.0;
                   %adding up the latitudonal values
                   ooo=ooo + double(oo)*t_scale+t_offset;
                   %how many latitudonal values did we use???
                   ninavg = ninavg + 1.0;
                   latitude(j);
               end
           end
           %make sure we don't divide by 0
           ninavg=max([1 ninavg]);
           %finally the latitudonal averrage
           t_altlong(k,i,d)=ooo/ninavg;
     end
end
end



t_altitude = double(newlevel);
```

```
%reducing the over latitude averraged values to only one time
t_altlongs=t_altlong(:,:,day);

%averaging over longitude, will be needed to substract the mean from the
%values
t_averrage_alt=mean(t_altlong,2);

%loop for substracting the mean over longitude
for d=1:1:double(size(time,1))-3,
for k=1:1:size(level,1),
for i=1:1:max(size(longitude)),
    t_d_altlong(k,i,d)=double(t_altlong(k,i,d))-double(t_averrage_alt(k,d));
end
end
end

%reducing to only one time
t_d_altlongs=t_d_altlong(:,:,day);

%reducing to only one level
t_d_altlongss_1=t_d_altlongs(IX(1),:,day);
t_d_altlongss_2=t_d_altlongs(IX(2),:,day);
t_d_altlongss_3=t_d_altlongs(IX(3),:,day);
t_d_altlongss_4=t_d_altlongs(IX(4),:,day);
```

Fast fourier transforming the prepared data

```
%windowing function to round the edges
window=hamming(144);
%rotating the windowing function to get it in the same dimensions as the
%actual data
```

```
window_t=rot90(window);

%windowing the data
t_window_1=t_d_altlongss_1.*window_t;
t_window_2=t_d_altlongss_2.*window_t;
t_window_3=t_d_altlongss_3.*window_t;
t_window_4=t_d_altlongss_4.*window_t;

%fourier transforming the data after having it filled up with zeros of length more than five
%times the dataslength and to a length of a potential of two
t_wave_1=abs(fft(t_window_1,1024));
t_wave_2=abs(fft(t_window_2,1024));
t_wave_3=abs(fft(t_window_3,1024));
t_wave_4=abs(fft(t_window_4,1024));

%shiftig the data to be able to recognize if the strongest peaks are the
%0´th one or different ones like 1 wave order
t_wave_1_shift=fftshift(t_wave_1);
t_wave_2_shift=fftshift(t_wave_2);
t_wave_3_shift=fftshift(t_wave_3);
t_wave_4_shift=fftshift(t_wave_4);

%shortening the spectrum to get it only one time
t_wave_1_s=t_wave_1(1:double(max(size(t_wave_1)))/2);
t_wave_2_s=t_wave_2(1:double(max(size(t_wave_2)))/2);
t_wave_3_s=t_wave_3(1:double(max(size(t_wave_3)))/2);
t_wave_4_s=t_wave_4(1:double(max(size(t_wave_4)))/2);
```

Plotting the results of the fft

```
%The plots give back the frequency on the x-axis and the fft of the PW
%amplitudes on the y-axis. The existing frequencies can be read out.
```

```
%scale for the x-axis
xaxis=9/64:9/64:72;

%plotting semilog fourier transformed and windowed data for all 4 levels
figure(1)
subplot(4,1,1); semilogx(xaxis,t_wave_1_s);
title('semilog fourier transformed and windowed data level 1');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');
subplot(4,1,2); semilogx(xaxis,t_wave_2_s);
title('semilog fourier transformed and windowed data level 2');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');
subplot(4,1,3); semilogx(xaxis,t_wave_3_s);
title('semilog fourier transformed and windowed data level 3');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');
subplot(4,1,4); semilogx(xaxis,t_wave_4_s);
title('semilog fourier transformed and windowed data level 4');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');

%plotting loglog fourier transformed and windowed data for all 4 levels
figure(2)
subplot(4,1,1); loglog(xaxis,t_wave_1_s);
title('loglog fourier transformed and windowed data level 1');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');
subplot(4,1,2); loglog(xaxis,t_wave_2_s);
title('loglog fourier transformed and windowed data level 2');
xlabel('Frequency(1/360°)');
```

```
ylabel('|Y(f)|');
subplot(4,1,3); loglog(xaxis,t_wave_3_s);
title('loglog fourier transformed and windowed data level 3');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');
subplot(4,1,4); loglog(xaxis,t_wave_4_s);
title('loglog fourier transformed and windowed data level 4');
xlabel('Frequency(1/360°)');
ylabel('|Y(f)|');

figure(3)
plot(t_d_altlongss_1);
```

8.2. amplitude_32_years_t

Isolating the amplitudes of the different wave orders from the PW

```
%After having found out which orders of planetary waves are existing in the
%Era 40 data from 1957/09/01 2002/08/31 by fouriertransforming the datasets
%from -40°S to -60°S it is now needed to extract the waveamplitudes.
%This is done to see if there are periodical fluctuations in the amplitudes
%that lead back to the qbo.

%This file takes the wave amplitudes and puts them into an own file
%amplitude_32_years_var_t

%in that file will be saved the following variables

%d_av_amp_t(year,wavenumber,level PW)   PW amplitudes after
%                                       taking off the mean
%f_amp_t(index,wavenumber, level PW)    the fourier transform of
%                                       d_av_amp_t, shortened to one
```

```
%                               period
%av_amp_t((year,wavenumber,level PW)    PW ampilitudes
%index (year,wavenumber,level PW)       the x-axis for possible plots of
%                               f_amp_t, given back are the
%                               frequencies in 1/year

%This step of taking the futurely needed variables and putting them into
%new files is needed to avoid a running of the current file since that
%takes at least one hour
```

loading in the ERA40 data set and creating links to the needed functions

```
addpath 'C:\Dokumente und Einstellungen\Tobias\Eigene Dateien\Studium\Bachelorthesis\Files_Jul_2000'



%Since the polar vortex is strongest at 60°S and it´s needed to find
%variations in the polar vortex, it will be most usefull to look at the
%edges



max_lat=-40;
min_lat=-60;

%Extracting the data and puttig it into matlab variables
S=netcdf('Data_t_July_1971_2002.nc');
[longitude,latitude,level,time,t]=S.VarArray.Data;
% now get the attributes
att_longitude   = S.VarArray(1,1).AttArray.Val;
att_latitude    = S.VarArray(1,2).AttArray.Val;
att_level       = S.VarArray(1,3).AttArray.Val;
att_time        = S.VarArray(1,4).AttArray.Val;
```

```
% Temperature (K)
[t_scale, t_offset, t_fill, t_missing, t_units, t_longname]        = S.VarArray(1,5).AttArray.Val;

% new format of net cdf files does not have level in pressure order, but
% instead in a sorted array, so, 1, 10, 100, 1000, 150, 2, 20 ...
% newlevel will be in sorted order, and IXX is the pointer into the array
[newlevel,IX]=sort(level,1);
```

preparing the ERA 40 data

```
%loop for averaging the data over the latitude between the min and max
%value.

%loop over time
for d=1:4:size(time),
    %loop over scale height
for k=1:1:size(level,1),
    %loop over longitudes (stepwith 2.5°)
     for i=1:1:size(longitude,1),
         %needed temporary variables
           oo=0;
           ooo = 0;
           ninavg = 0.0;
           %averaging loop over latitude between min and max
           for j=1:1:size(latitude,1),
               %make shure we only averrage between min and max latitude
               if((latitude(j) >= min_lat) && (latitude(j)<=max_lat))
                   %values with a period of 6 hours are not needed and
                   %influenced by the sun, what leads to failures,
                   %therefore averaging over 1 day period

oo=(double(t(d,IX(k),j,i))+double(t(d+1,IX(k),j,i))+double(t(d+2,IX(k),j,i))+double(t(d+3,IX(k),j,i)))/4.0;
```

```
                %adding up the latitudonal values
                ooo=ooo + double(oo)*t_scale+t_offset;
                %how many latitudonal values did we use???
                ninavg = ninavg + 1.0;
                latitude(j);
            end
        end
        %make sure we don't divide by 0
        ninavg=max([1 ninavg]);
        %finally the latitudonal averrage
        t_altlong(k,i,d)=ooo/ninavg;
    end
end
end

clear d k i oo ooo ninavg j

t_altitude = double(newlevel);
%averaging over longitude, will be needed to substract the mean from the
%values
t_averrage_alt=mean(t_altlong,2);

%loop for substracting the mean over longitude
for d=1:1:double(size(time,1))-3,
for k=1:1:size(level,1),
for i=1:1:max(size(longitude)),
   t_d_altlong(k,i,d)=double(t_altlong(k,i,d))-double(t_averrage_alt(k,d));
end
end
end

clear d k i
```

Levenberg Marquardt fitting the ERA 40 data to extract the PW amplitudes

```
%As indicated in "t_fft_apodization_average_over_latitude_July_2000.m" is
%assumed, that the variation in t along a latitudonal circle obeys mostly
%planetary waves of wavebumber 1,2,3, maybe 4,5,6.
%It will now be tried to fit a function: x1 + x2*cos(2*pi*longitude/360) +
%x3*sin(2*pi*longitude/360) + x4*cos(2*pi*longitude/180) +
%x5*sin(2*pi*longitude/180) + x6*cos(2*pi*longitude/120) +
%x7*sin(2*pi*longitude/120) to the data in order to achieve the amplitudes
%of the different orders of planetary waves.
%This will be done for one time a day over each dayy of each July of each
%year from 1971 to 2002



%loop over time
for day=1:4:(size(time)-3),
%reducing to only one time
t_d_altlongs=t_d_altlong(:,:,day);
for height=1:1:max(size(level)),
%reducing to only one level
t_d_altlongss=t_d_altlongs(height,:);

%rotating the array to get them into the same size as longitude
t_d_altlongsst=rot90(t_d_altlongss);

%defining the function to which the data will be fitted
Eq=@(x)
x(1)+x(2)*cos(2*pi*longitude/360)+x(3)*sin(2*pi*longitude/360)+x(4)*cos(2*pi*longitude/180)+x(5)*sin(2*pi*longitude/180)+x(6)*cos(2*pi*longitude/120)+x(7)*sin(2*pi*longitude/120)+x(8)*cos(2*pi*longitude/90)+x(9)*sin(2*pi*longitude/90)+x(10)*cos(2*pi*longitude/72)+x(11)*sin(2*pi*longitude/72)+x(12)*cos(2*pi*longitude/60)+x(13)*sin(2*pi*longitude/60)-t_d_altlongsst;
```

```
%giving a first guess of the variables x(1),...,x(13) into the array
%x(0)
x0=[0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
%the fitting algorythmus
[x,ssq,cnt] = LMFnlsq(Eq,x0);

%test to find out if the fitting worked, if it didn´t it will give back the
%number 666 (as soon as i know devil´s nuber, so reeeeally bad)
if ssq/sqrt(sum(t_d_altlongsst.*t_d_altlongsst))>0.01
    warning('The fitting algorithm didn´t work correctly, propably t_d_altlongsst needs to be divided by a power of 10 (don´t forget to multiplicate amp and amp_phase ba the same number again)')
end

figure(day)
subplot(max(size(level)),1,height); plot(longitude,t_d_altlongsst, longitude,Eq(x)+t_d_altlongsst,'r'), grid;
title('Levenberg Marquardt Fit');
xlabel('longitude');
ylabel('Y(f)');

%calculating the amplitudes for waveorders 1,...,6
amp_order1=(((x(2))^2+(x(3))^2)^(1/2));
amp_order2=((x(4))^2+(x(5))^2)^(1/2);
amp_order3=((x(6))^2+(x(7))^2)^(1/2);
amp_order4=((x(8))^2+(x(9))^2)^(1/2);
amp_order5=((x(10))^2+(x(11))^2)^(1/2);
amp_order6=((x(12))^2+(x(13))^2)^(1/2);

%calculating the phases for waveorders 1,...,6
phase_order1=atan((x(2))/(x(3)));
phase_order2=atan((x(4))/(x(5)));
phase_order3=atan((x(6))/(x(7)));
phase_order4=atan((x(8))/(x(9)));
```

```
phase_order5=atan((x(10))/(x(11)));
phase_order6=atan((x(12))/(x(13)));

%putting the amplitude and the phase in an array of the form
%amp_phase_ee(amp=1 phase=2,order 1-6,height 1-4,day 1-end)
%and another one
%of the form
%amp_ee(amp cos=1 amp sin=2, order 1-6, height 1-4, day 1-end (992))
amp_phase_ss=double([amp_order1 amp_order2 amp_order3 amp_order4 amp_order5 amp_order6;
phase_order1 phase_order2 phase_order3 phase_order4 phase_order5 phase_order6]);
amp_ss=double([x(2) x(4) x(6) x(8) x(10) x(12); x(3) x(5) x(7) x(9) x(11) x(13)]);
amp_phase_s(:,:,height)=amp_phase_ss;
amp_s(:,:,height)=amp_ss;
end
amp_phase(:,:,:,(day+3)/4)=amp_phase_s;
amp(:,:,:,(day+3)/4)=amp_s;
end

clear phase_order1 phase_order2 phase_order3 phase_order4 amp_order1 amp_order2 amp_order3 amp_order4
clear amp_phase_s amp_phase_ss day height t_d_altlongsst t_d_altlongss or_d_altlongs x0 x ssq cnt
clear height day
```

recomposing the PW seperated in waveorders

```
%creating waves with one waveorder of the form
%cos(longitude*2*pi/360*waveorder+phase)
% Here the disadvantage is, that the amplitude is, that the calculation
% A=((x1)^2+(x2)^2)^1/2 allways leads to positive amplitudes, what is not
% consistent with the reall amplitudes which are sometimes negatively.
%This leads to a failure in phase of 180°.
%Problematically for Hovmöllerplots, not a problem for further
%calculations, since there will anyway only be needed the abbsolute values
```

```
%of the amplitudes.
for waveorder=1:1:6,
   for day=1:1:992
      for level=1:1:4,
         for long=1:1:144

wave_wo_amp_phase(day,long,waveorder,level)=amp_phase(1,waveorder,level,day)*cos(2*pi*longitude(long,1)/(360/waveorder)+amp_phase(2,waveorder,level,day));
         end
         end
   end
end

clear waveorder day level long

%creating waves with one waveorder of the form
%cos(longitude*2*pi/360*waveorder) + sin(longitude*2*pi/360*waveorder) what
%solves the above mentioned problem of the 180° phase or amplitude
%failures.
for waveorder=1:1:6
   for level=1:1:4
      for days=1:1:992
         for long=1:1:144

wave_wo_amp(days,long,waveorder,level)=amp(1,waveorder,level,days)*cos(2*pi*longitude(long,1)/(360/waveorder))+amp(2,waveorder,level,days)*sin(2*pi*longitude(long,1)/(360/waveorder));
         end
      end
   end
end

clear waveorder level days long
```

```
days=1:1:992;
long=1:1:144;
```

Preparing the amp_phase data (averaging over months)

```
%Eventhough the data of a cosine + a sine are better to reconstruct the
%wavepattern over longitude the cosine with phase is better usable for the
%coming analysis since one has only one amplitude and the phase is not
%important any more

%Averaging the amplitudes over each month and putting the data in yearly
%dependence (1 Datapoint per year, July).
%Whereby the sign of the amplitudes is not important.
   for level=1:1:4
      for waveorder=1:1:6
         for days=1:31:(max(size(wave_wo_amp(:,1,1,1))))
            aaa=0;
            for ddays=1:1:30
               aa=amp_phase(1,waveorder,level,days+ddays);
               aaa=aaa+aa;
            end
            av_amp((days+30)/31,waveorder,level)=aaa/31;
         end
      end
   end

clear aa aaa ddays days level waveorder

%Taking the mean from av_amp
for level=1:1:4
   for waveorder=1:1:6
```

```
        av_amp_mean=mean(av_amp(:,waveorder,level),1);
        for years=1:1:32
            d_av_amp(years,waveorder,level)=av_amp(years,waveorder,level)-av_amp_mean;
        end
    end
end

clear years waveorder level av_amp_mean
```

First analysis of the PW amplitude data

```
%Taking the fourier transform of the amplitudes in order to find out, which
%frequencies do exist

%windowing function to round the edges
window=hamming(32);
%rotating the windowing function to get it in the same dimensions as the
%actual data
for level=1:1:4
    for waveorder=1:1:6
        amp_window=d_av_amp(:,waveorder,level).*window;
        amp_window_t=rot90(amp_window);
        f_amp_l=rot90(abs(fft(amp_window_t,1024)));
        f_amp_s=f_amp_l(1:(double(max(size(f_amp_l)))/2));
        f_amp(:,waveorder,level)=f_amp_s;
    end
end

%index array, is needed to plot the x-axis of the fouriertransform
%In this case the index gives back the existing frequencies
index=rot90(1/2/512:1/2/512:1/2);
```

```
d_av_amp_t=d_av_amp;
f_amp_t=f_amp;
av_amp_t=av_amp;
```

clearing all variables but the on the beginning mentioned ones and saving them into a new data file

```
%clearing
clearvars -except d_av_amp_t f_amp_t av_amp_t index

%saving
save amplitude_32_years_var_t
```

8.3. QBO

Comparisson between the qbo data and the PW amplitudes

```
%In this file is compared the qbo and the planetary waves amplitudes in
%order to learn about the crossconvolutioin and the resulting phases and
%frequencies between both datasets
%The main idea is to find out about, which level of the qbo influences the
%PW most

%Often loops are made for the variables (u,z,o3,t) of the PW amplitudes
%seperabely, when this occurs onl the first loop is commented
```

loading in the data

```
addpath 'C:\Dokumente und Einstellungen\Tobias\Eigene Dateien\Studium\Bachelorthesis\Files_Jul_2000'
import_qbo_data u_profile_extres.txt
load amplitude_32_years_var_t
load amplitude_32_years_var_o3
load amplitude_32_years_var_z
```

```
load amplitude_32_years_var_u
```

preparing the qbo data

```
%in the first colum of the QBO Data used to be written down the number of years, since that
%number is not needed in any calculation, the colum becomes cut off
for i=2:1:max(size(data(1,:))),
    qbo_s(:,i-1)=data(:,i);
end
clear i data



%It is only required to get the QBO Data from July of each year
%Therefore a new array is made with annual data from July
for i=7:12:max(size(qbo_s)),
    qbo_july_l((i+5)/12,:)=qbo_s(i,:);
end
clear i qbo_s

%The variable depending data are reaching over a shorter time range than
%the QBO data, wherefore it is needed to shorten these data in order to
%achieve the same length for both.
%The Data are shortened to make them both reach from 1971-2002
%QBO former reyched from 1953 to 2004
for i=19:1:50,
    qbo_july(i-18,:)=qbo_july_l(i,:);
end
clear i qbo_july_l

%checking if some default values are existing in the data
%if yes, "default value" will be given back
for i=1:1:min(size(qbo_july)),
```

```
    for j=1:1:max(size(qbo_july)),
        if qbo_july(j,i)==99.9,
            warning('there is an existing default value in the roar data of QBO')
        end
    end
end
clear i j

%Taking of the mean from the QBO data
a=0;
aa=0;
b=0;
for i=1:1:max(size(qbo_july(:,1)))
    for j=1:1:max(size(qbo_july(1,:)))
        a=qbo_july(i,j);
        aa=aa+a;
        b=b+1;
    end
end
mean_qbo_july=aa/b;
for i=1:1:max(size(qbo_july(:,1)))
    for j=1:1:max(size(qbo_july(1,:)))
        qbo_july(i,j)=qbo_july(i,j)-mean_qbo_july;
    end
end
clear a aa b mean_qbo_july
```

crossconvoluting the prepared qbo data and the PW data

```
%Taking the QBO Data and cross corellating them with the temperature,
%ozone, scale height and wind velocity data
%Putting these data into an 4D-Array of the form
```

```
%x_cov_variable(lag[time], level QBO (1-19), waveorder (1-6), level
%variable (1-4))

%loop over the pressure levels of the QBO data
for i=1:1:max(size(qbo_july(1,:))),
    %loop over the wave numbers of the PW data
    for j=1:1:max(size(d_av_amp_t(1,:,1))),
        %loop over the pressure levels of the PW data
        for k=1:1:max(size(d_av_amp_t(1,1,:))),
            %isolating the annual PW data for each pressure level and each
            %wave order
            d_av_amp_t_e=[d_av_amp_t(:,j,k)];
            %taking the crosscovariance of the PW data with the annual QBO
            %data
            xcov_t_s=xcov(qbo_july(:,i),d_av_amp_t_e,'coeff');
            %Putting the crosscovariance into the above mentioned array
            xcov_t(i,:,j,k)=xcov_t_s(:);
        end
    end
end
%clearing not needed variables
clear xcov_t_s i j k d_av_amp_t_e



for i=1:1:max(size(qbo_july(1,:))),
    for j=1:1:max(size(d_av_amp_o3(1,:,1))),
        for k=1:1:max(size(d_av_amp_o3(1,1,:))),
            d_av_amp_o3_e=[d_av_amp_o3(:,j,k)];
            xcov_o3_s=xcov(qbo_july(:,i),d_av_amp_o3_e,'coeff');
            xcov_o3(i,:,j,k)=xcov_o3_s(:);
        end
    end
```

```
end
clear xcov_o3_s i j k d_av_amp_o3_e


for i=1:1:max(size(qbo_july(1,:))),
    for j=1:1:max(size(d_av_amp_u(1,:,1))),
        for k=1:1:max(size(d_av_amp_u(1,1,:))),
           d_av_amp_u_e=[d_av_amp_u(:,j,k)];
            xcov_u_s=xcov(qbo_july(:,i),d_av_amp_u_e,'coeff');
            xcov_u(i,:,j,k)=xcov_u_s(:);
        end
    end
end
clear xcov_u_s i j k d_av_amp_u_e


for i=1:1:max(size(qbo_july(1,:))),
    for j=1:1:max(size(d_av_amp_z(1,:,1))),
        for k=1:1:max(size(d_av_amp_z(1,1,:))),
            d_av_amp_z_e=[d_av_amp_z(:,j,k)];
            xcov_z_s=xcov(qbo_july(:,i),d_av_amp_z_e,'coeff');
            xcov_z(i,:,j,k)=xcov_z_s(:);
        end
    end
end
clear xcov_z_s i j k d_av_amp_z_e


%creating arrays that fit the dimensions of the plottable arrays to get the
%axis in the right size

%Time lag is needed for plotting cross covariances
```

```
time_lag=rot90(rot90(rot90(-31:1:31)));
%the QBO levels ascending in pressure, descending in height
level_QBO=[3; 4; 5; 6; 8; 10; 12; 15; 20; 25; 30; 35; 40; 45; 50; 60; 70; 80; 90]';
%PW level ascending in pressure, descending in height
level_PW=[1; 10; 50; 100];



%plotting the covariances into figures
%The number of the figure divided by 6, the resulting integer is the
%levelnumber [1,2,3,4]==[1,10,50,100] and the resulting rest is the
%wavebumber

% for j=1:1:max(size(xcov_t(1,1,1,:))),
%     for i=1:1:max(size(xcov_t(1,1,:,1))),
%         k=((j-1)*6+i);
%         figure(k)
%         contourf(time_lag, level_QBO, xcov_t(:,:,i,j))
%         colorbar
%     end
% end
% clear i j k
```

making an fft of the xc resluts

```
%To find out which effects influence what it is needed to find out about
%the existing frequencies in the xc data
%the results will be given back in an array of the form:
%fft_variable(freq,level QBO, waveorder, level variable)

%therefore first of all the dimensions have to be switched
for i=1:1:max(size(xcov_t(:,1,1,1))),
    for j=1:1:max(size(xcov_t(1,:,1,1))),
```

```
        for k=1:1:max(size(xcov_t(1,1,:,1))),
            for l=1:1:max(size(xcov_t(1,1,1,:))),
            xcov_t_t(j,i,k,l)=xcov_t(i,j,k,l);
            xcov_t_o3(j,i,k,l)=xcov_o3(i,j,k,l);
            xcov_t_u(j,i,k,l)=xcov_u(i,j,k,l);
            xcov_t_z(j,i,k,l)=xcov_z(i,j,k,l);
            end
        end
    end
end
clear i j k l

%the fourier transform of the planetary wave data puting them into the form
%fft_variable (frequency,level QBO, waveorder, level planetary wave)

%loop over the QBO levels
for i=1:1:max(size(xcov_t_t(1,:,1,1))),
    %loop over the waveorders
    for j=1:1:max(size(xcov_t_t(1,1,:,1))),
        %loop over the PW levels
        for k=1:1:max(size(xcov_t_t(1,1,1,:))),
            %fast fourier transforming the crosscovariances after padding
            %them with zeors to a length that is a power of 2 and at least
            %4 times the length of the data
            fft_t_l=fft(xcov_t_t(:,i,j,k),512);
            %shortening the fft from 2 to 1 harmonics
            fft_t(:,i,j,k)=fft_t_l(1:(double(max(size(fft_t_l))/2)));
        end
    end
end
%clearing not any more needed variables
clear i j k fft_t_l
```

```
for i=1:1:max(size(xcov_t_o3(1,:,1,1))),
    for j=1:1:max(size(xcov_t_o3(1,1,:,1))),
        for k=1:1:max(size(xcov_t_o3(1,1,1,:))),
            fft_o3_l=fft(xcov_t_o3(:,i,j,k),512);
            fft_o3(:,i,j,k)=fft_o3_l(1:(double(max(size(fft_o3_l))/2)));
        end
    end
end
clear i j k fft_t_l

for i=1:1:max(size(xcov_t_z(1,:,1,1))),
    for j=1:1:max(size(xcov_t_z(1,1,:,1))),
        for k=1:1:max(size(xcov_t_z(1,1,1,:))),
            fft_z_l=fft(xcov_t_z(:,i,j,k),512);
            fft_z(:,i,j,k)=fft_z_l(1:(double(max(size(fft_z_l))/2)));
        end
    end
end
clear i j k fft_t_l

for i=1:1:max(size(xcov_t_u(1,:,1,1))),
    for j=1:1:max(size(xcov_t_u(1,1,:,1))),
        for k=1:1:max(size(xcov_t_u(1,1,1,:))),
            fft_u_l=fft(xcov_t_u(:,i,j,k),512);
            fft_u(:,i,j,k)=fft_u_l(1:(double(max(size(fft_u_l))/2)));
        end
    end
end
clear i j k fft_t_l
```

```
%creating the freyuency axis, which is needed to be able to plot the fft,
%the max frequency is the nyquist frequency, so half the sampling frequency
index=(1/2/256:1/2/256:1/2)';
index2=1./index;


%plotting the fft into a contourplot one per figure, recognice, that the
%axes are switched!!!
%The number of the figure divided by 6, the resulting integer is the
%levelnumber [1,2,3,4]==[1,10,50,100] and the resulting rest is the
%wavebumber

% for j=1:1:max(size(fft_t(1,1,1,:))),
%     for i=1:1:max(size(fft_t(1,1,:,1))),
%         k=((j-1)*6+i);
%         figure(k)
%         contourf(level_QBO, index, abs(fft_t(:,:,i,j)))
%         colorbar
%     end
% end
% clear i j k


%plotting the fft into a "normal" plot, one per figure
%The number of the figure divided by 6, the resulting integer is the
%levelnumber [1,2,3,4]==[1,10,50,100] and the resulting rest is the
%wavebnumber

% for j=1:1:max(size(fft_t(1,1,1,:))),
%     for i=1:1:max(size(fft_t(1,1,:,1))),
%         k=((j-1)*6+i);
%         figure(k)
```

```
%         plot(index,abs(fft_t(:,7,i,j)))
%     end
% end
% clear i j k
%
```

spline interpolating the xc dataset

```
%spline interpolating the crosscovariance data and putting them into a
%variable of the form spline(xx,level QBO, wavenumber, level PW)
%to be able to lokate the peaks of the QBO in the PW Data more easily

%Therefore first must 2 new arrays be created:
%one with the original length of xcov_t_variable(:,1,1,1)
%one with the wished length of spline_variable(:,1,1,1) what is xx

%The array with the original length of xcov_t_variable(:,1,1,1)
x=1:1:63;
%The array with the wished length of spline_variable(:,1,1,1)
xx=(0.5^5):(0.5^5):63;

%loop over the QBO pressure levels
for i=1:1:max(size(xcov_t_t(1,:,1,1)))
    %loop over the PW waveorders
    for j=1:1:max(size(xcov_t_t(1,1,:,1)))
        %loop over the PW pressure levels
        for k=1:1:max(size(xcov_t_t(1,1,1,:)))
            %taking the data into a local variable
            y=xcov_t_t(:,i,j,k);
            %spline interpolating them
            yy=spline(x,y,xx);
            %and putting them into an array of the above mentioned form
```

```
            spline_t(:,i,j,k)=yy(1,:);
        end
    end
end
%clearing not any more needed variables
clear i j k  y yy

for i=1:1:max(size(xcov_t_o3(1,:,1,1)))
    for j=1:1:max(size(xcov_t_o3(1,1,:,1)))
        for k=1:1:max(size(xcov_t_o3(1,1,1,:)))
            y=xcov_t_o3(:,i,j,k);
            yy=spline(x,y,xx);
            spline_o3(:,i,j,k)=yy(1,:);
        end
    end
end
clear i j k y yy

for i=1:1:max(size(xcov_t_u(1,:,1,1)))
    for j=1:1:max(size(xcov_t_u(1,1,:,1)))
        for k=1:1:max(size(xcov_t_u(1,1,1,:)))
            y=xcov_t_u(:,i,j,k);
            yy=spline(x,y,xx);
            spline_u(:,i,j,k)=yy(1,:);
        end
    end
end
clear i j k y yy

for i=1:1:max(size(xcov_t_z(1,:,1,1)))
    for j=1:1:max(size(xcov_t_z(1,1,:,1)))
        for k=1:1:max(size(xcov_t_z(1,1,1,:)))
```

```
            y=xcov_t_z(:,i,j,k);
            yy=spline(x,y,xx);
            spline_z(:,i,j,k)=yy(1,:);
        end
    end
end
clear i j k y yy x xx
```

creating lomb-scargle plots of the xc data

```
%The fastlomb function is a function which makes in principal the same as a
%fft, but that it gives back a significance level for the peaks as well.
%This is done by fitting harmonics to the datasets and looking at standard
%deviations. If graphs are required, change fastlomb(x,t,o,...) to
%fastlomb(x,t,k,...) This will give ack the same graph structure as above
%Here is only taken into akkount the 40 hPa pressure level which is level
%#7

%loop over the QBO pressure levels
for i=1:1:max(size(xcov_t_t(1,1,:,1))),
    %loop over the PW waveorders
    for j=1:1:max(size(xcov_t_t(1,1,1,:))),
        %taking the data into a local variable
        x=xcov_t_t(:,7,i,j);
        %creating a new local variable that has the length of the time lag
        %array for the crosscovariance
        %crosscovariance
        t=rot90(1:1:63);
        %creating a new array that sorts graphs into the old routine
        %devide by 6, integers are level, rest is wave order
        k=((j-1)*6+i);
        %executing the fastlomb function
```

```
        %x given above; t given above; figure k; default for hifag and
        %ofag; additional significance level of 0.9999
        %in this case the function is only used to achieve a graph and not
        %to continue working with the results since they do allready exist
        %from the fft
        fastlomb(x,t,0,1,4,0.9999);
        %clearing the variables to avoide mistakes in the loop
        clear ans x t k
    end
end

for i=1:1:max(size(xcov_t_o3(1,1,:,1))),
    for j=1:1:max(size(xcov_t_o3(1,1,1,:))),
        x=xcov_t_o3(:,7,i,j);
        t=rot90(1:1:63);
        k=((j-1)*6+i);
        fastlomb(x,t,0,1,4,0.9999);
        clear ans x t k
    end
end

for i=1:1:max(size(xcov_t_z(1,1,:,1))),
    for j=1:1:max(size(xcov_t_z(1,1,1,:))),
        x=xcov_t_z(:,7,i,j);
        t=rot90(1:1:63);
        k=((j-1)*6+i);
        fastlomb(x,t,0,1,4,0.9999);
        clear ans x t k
    end
end

for i=1:1:max(size(xcov_t_u(1,1,:,1))),
```

```
    for j=1:1:max(size(xcov_t_u(1,1,1,:))),
        x=xcov_t_u(:,7,i,j);
        t=rot90(1:1:63);
        k=((j-1)*6+i);
        fastlomb(x,t,0,1,4,0.9999);
        clear ans x t k
    end
end
```

Does averaging about all the PW amplitudes cancell out noise???

```
%Adding up all the planetary wave amplitudes in order to find out if that
%cancells out noise.
%Herefore all steps (crossconvoluting, fouriertransformating, spline
%interpolating and normalizing) are made in one loop.

%The array with the original length of xcov_t_variable(:,1,1,1)
x=1:1:63;
%The array with the wished length of spline_variable(:,1,1,1)
xx=1:(0.5^5):63;

%loop over the QBO pressure levels
for i=1:1:max(size(qbo_july(1,:))),
    %creating lovcal variables
    a=0;
    aa=0;
    b=0;
    %loop over the waveorder of the PW data
    for j=1:1:max(size(d_av_amp_t(1,:,1))),
        %loop over the pressure levels of the PW amplitudes
        for k=1:1:max(size(d_av_amp_t(1,1,:))),
            %putting the PW amplitudes into a local variable
```

```
            a=d_av_amp_t(:,j,k);
            %adding up all the PW amplitudes for each pressure level and
            %for each waveorder
            aa=aa+a;
            %counting up the counter that will be the divisor for the
            %quotient average PW amplitudes
            b=b+1;
        end
    end
    %taking the crosscovariance of the average PW amplitude with the QBO
    %amplitude
    xcov_a_t_s=xcov(qbo_july(:,i),aa/b,'coeff');
    %putting that result into a final variable
    xcov_a_t(:,i)=xcov_a_t_s;
    %creating a new local variable that of the crossvariance
    y=xcov_a_t(:,i);
    %spline interpolating the crosscovariance in order to get a higher
    %resolution graph
    yy=spline(x,y,xx);
    %putting the spline interpolation into a new variable
    spline_a_t(:,i)=yy(1,:);
    %fourier transforming the crosscovariance after padding it
    fft_a_t_l=fft(xcov_a_t(:,i),512);
    %shortening the fft from two harmonics to one
    fft_a_t_u_c=fft_a_t_l(1:(double(max(size(fft_a_t_l))/2)));
    %taking the absolute value of the fft
    fft_a_t_u=abs(fft_a_t_u_c);
    %and putting that into a final variable
    fft_a_t(:,i)=fft_a_t_u/max(max(fft_a_t_u));
end
clear a aa b j i k xcov_a_t_s c fft_a_t_l y yy
```

```
for i=1:1:max(size(qbo_july(1,:))),
    a=0;
    aa=0;
    b=0;
    for j=1:1:max(size(d_av_amp_o3(1,:,1))),
        for k=1:1:max(size(d_av_amp_o3(1,1,:))),
            a=d_av_amp_o3(:,j,k);
            aa=aa+a;
            b=b+1;
        end
    end
    xcov_a_o3_s=xcov(qbo_july(:,i),aa/b,'coeff');
    xcov_a_o3(:,i)=xcov_a_o3_s;
    y=xcov_a_o3(:,i);
    yy=spline(x,y,xx);
    spline_a_o3(:,i)=yy(1,:);
    fft_a_o3_l=fft(xcov_a_o3(:,i),512);
    fft_a_o3_u_c=fft_a_o3_l(1:(double(max(size(fft_a_o3_l))/2)));
    fft_a_o3_u=abs(fft_a_o3_u_c);
    fft_a_o3(:,i)=fft_a_o3_u/max(max(fft_a_o3_u));
end
clear a aa b j i k xcov_a_o3_s c fft_a_o3_l y yy

for i=1:1:max(size(qbo_july(1,:))),
    a=0;
    aa=0;
    b=0;
    for j=1:1:max(size(d_av_amp_z(1,:,1))),
        for k=1:1:max(size(d_av_amp_z(1,1,:))),
            a=d_av_amp_z(:,j,k);
            aa=aa+a;
            b=b+1;
```

```
        end
    end
    xcov_a_z_s=xcov(qbo_july(:,i),aa/b,'coeff');
    xcov_a_z(:,i)=xcov_a_z_s;
    y=xcov_a_z(:,i);
    yy=spline(x,y,xx);
    spline_a_z(:,i)=yy(1,:);
    fft_a_z_l=fft(xcov_a_z(:,i),512);
    fft_a_z_u_c=fft_a_z_l(1:(double(max(size(fft_a_z_l))/2)));
    fft_a_z_u=abs(fft_a_z_u_c);
    fft_a_z(:,i)=fft_a_z_u/max(max(fft_a_z_u));
end
clear a aa b j i k xcov_a_z_s c fft_a_z_l y yy

for i=1:1:max(size(qbo_july(1,:))),
    a=0;
    aa=0;
    b=0;
    for j=1:1:max(size(d_av_amp_u(1,:,1))),
        for k=1:1:max(size(d_av_amp_u(1,1,:))),
            a=d_av_amp_u(:,j,k);
            aa=aa+a;
            b=b+1;
        end
    end
    xcov_a_u_s=xcov(qbo_july(:,i),aa/b,'coeff');
    xcov_a_u(:,i)=xcov_a_u_s;
    y=xcov_a_u(:,i);
    yy=spline(x,y,xx);
    spline_a_u(:,i)=yy(1,:);
    fft_a_u_l=fft(xcov_a_u(:,i),512);
    fft_a_u_u_c=fft_a_u_l(1:(double(max(size(fft_a_u_l))/2)));
```

```
    fft_a_u_u=abs(fft_a_u_u_c);
    fft_a_u(:,i)=fft_a_u_u/max(max(fft_a_u_u));
end
clear a aa b j i k xcov_a_u_s c fft_a_u_l y yy
clear x xx
```

comparing qbo and averaged PW frequencyspectra

```
%Here is tried to compare the fft of the QBO data with the fft of the
%planetary wave data. The question is, how similar they look. That is asked
%in order to find out what makes direct effects on the strength of the
%planetary waves.
%With this question  is tried to give a reason for the extremely small
%ozone hole in spring 2002

for i=1:1:max(size(qbo_july(1,:))),
    xcov_qbo_july=xcov(qbo_july(:,i),qbo_july(:,i),'coeff');
    fft_qbo_u_c_l=fft(xcov_qbo_july,512);
    %shortening to one harmonic (unnormalized, clomplex)
    fft_qbo_u_c=fft_qbo_u_c_l(1:(double(max(size(fft_qbo_u_c_l))/2)));
    %taking the absolute value
    fft_qbo_u=abs(fft_qbo_u_c);
    %normalizing
    fft_QBO(:,i)=fft_qbo_u/max(fft_qbo_u);
end
clear fft_qbo_l i

index2=1./index;

figure(1)
hold on
semilogx(index2,abs(fft_QBO(:,7)),'g')
```

```
semilogx(index2,abs(fft_a_t(:,7)),'r')
hold off
```

8.4. phase_between_QBO_andPW

Which levels of the QBO influence the PW amplitudes most?

```
%This programm is written to find the heights of the QBO that affect the PW
%By looking at figure(2) one can see, that left and right from zero is the
%expected pattern but not at zero, where it is needed. Therefore first
%needs to be made a levenberg marquardt fit to find the harmonics and then
%reconstructed the pattern without the higher wavelengths, which propably
%make the pattern disappear at zero.
%To find the significant peaks first a Lomb-Scargle fit will be done.
```

Loading in the data and creating links to the needed programms

```
addpath 'C:\Dokumente und Einstellungen\Tobias\Eigene Dateien\Studium\Bachelorthesis\Files_Jul_2000'
load xcov_pw_QBO_average_amplitudes

% The loaded in variables are:
% fft_QBO       the fourier transform of the QBO
% fft_a_o3      the fourier transform of the crosscovariance of the qbo and
%               the (over each level of height and wavenumber) averaged PW
%               amplitudes (for the measured variable o3)
% fft_a_t       (for the measured variable t)
% fft_a_u       (for the measured variable u)
% fft_a_z       (for the measured variable z)
% index         the x-axis of the fft-plots (assumed 256 datapoints)
% index2        1/index
% level_PW      PW level ascending in pressure, descending in height
```

```
%             level_PW=[1; 10; 50; 100];
% level_QBO    the QBO levels ascending in pressure, descending in height
%             level_QBO=[3; 4; 5; 6; 8; 10; 12; 15; 20; 25; 30; 35; 40; 45; 50; 60; 70; 80; 90]';
% spline_a_o3  spline interpolation of the (over each level of height and
%             wavenumber) PW amplitudes (for the measured variable o3)
% spline_a_t   (for the measured variable t)
% spline_a_u   (for the measured variable u)
% spline_a_z   (for the measured variable z)
% time_lag     needed as the x-axis of figure(2), stepwith 1
%xcov_a_o3     the crosscovariance of the qbo and
%             the (over each level of height and wavenumber) averaged PW
%             amplitudes (for the measured variable o3)
% xcov_a_t     (for the measured variable t)
% xcov_a_u     (for the measured variable u)
% xcov_a_z     (for the measured variable z)
```

plotting the xcov and the fft (of the qbo and the av. PW amplitudes)

```
%time lag in the xcov of the qbo and the over all levels and wavenumbers
%averaged PW amplitudes, stepwith 0.1
time_lag_2=-31:0.1:31;

%fft_QBO and fft_a_t plotted in the same graph over index 2. It shows that
%both datasets content allmost the same amplitudes
figure(1)
hold on
semilogx(index2,abs(fft_QBO(:,7)),'g')
semilogx(index2,abs(fft_a_t(:,7)),'r')
hold off

% plotting xcov_a_t in a contourplot. One can see, that because of some
```

```
% effects the needed plot above zero time-lag is not very sharp
%There are propably lower beating frequencies existent, which forst have to
%be seperated out
figure(2)
contourf(time_lag',level_QBO',xcov_a_t')
```

Lomb scargle of xcov_a_t to filter out the significant peaks

```
%fastlomb of xcov_a_t to find ojut the significant peaks (u,z,o3 were
%resigned, since they show the same peaks but with more noise)
%(the plots are disabled in order to save time, if it is resigned to see
%them change fastlomb(x,t,0,1,4); to fastlomb(x,t,k,1,4);
for i=1:1:max(size(xcov_a_t(1,:)))
    x=xcov_a_t(:,i);
    t=rot90(1:1:63);
    k=i+2;
    fastlomb(x,t,0,1,4);
end
clear i k t
```

Levenberg-Marquardt fitting the xcov_a_t data with different number of peaks

```
%(sifnificance level set differently)
%As one can see only the last version with only one peak filters out the
%qbo without any longer timescale influence.

for i=1:1:max(size(xcov_a_t(1,:)))


    xcov_a_t_s=xcov_a_t(:,i)*1e-003;


    Eq=@(x)
x(1)+x(2)*cos(2*pi*time_lag*0.430)+x(3)*sin(2*pi*time_lag*0.430)+x(4)*cos(2*pi*time_lag*0.485)+x(5)*si
```

```
n(2*pi*time_lag*0.485)+x(6)*cos(2*pi*time_lag*0.400)+x(7)*sin(2*pi*time_lag*0.400)+x(8)*cos(2*pi*time_lag*0.370)+x(9)*sin(2*pi*time_lag*0.370)-xcov_a_t_s;
    x0=[0;0;0;0;0;0;0;0;0];

    [x,ssq,cnt]=LMFnlsq(Eq,x0);

    if ssq/sqrt(sum(xcov_a_t_s.*xcov_a_t_s))>0.01
       warning('xcov_a_t contends to big values, the Levenberg-Marquardt fit will fail')
    end

    amp_ss=double([x(2) x(4) x(6) x(8); x(3) x(5) x(7) x(9)]);
    amp(:,:,i)=amp_ss*1e003;
    start(:,i)=x(1);
end

clear  x x0 ssq cnt Eq xcov_a_t_s amp_ss

for i=1:1:max(size(amp(1,1,:)))

x=start(1,i)+amp(1,1,i)*cos(2*pi*time_lag*0.430)+amp(2,1,i)*sin(2*pi*time_lag*0.430)+amp(1,2,i)*cos(2*pi*time_lag*0.485)+amp(2,2,i)*sin(2*pi*time_lag*0.485)+amp(1,3,i)*cos(2*pi*time_lag*0.400)+amp(2,3,i)*sin(2*pi*time_lag*0.400)+amp(1,4,i)*cos(2*pi*time_lag*0.370)+amp(2,4,i)*sin(2*pi*time_lag*0.370);
    xcov_a_t_qbo(:,i)=x;
end

clear x

for i=1:1:max(size(xcov_a_t(1,:)))

    xcov_a_t_s=xcov_a_t(:,i)*1e-003;
```

```
    Eq=@(x)
x(1)+x(2)*cos(2*pi*time_lag*0.460)+x(3)*sin(2*pi*time_lag*0.460)+x(4)*cos(2*pi*time_lag*0.400)+x(5)*si
n(2*pi*time_lag*0.400)+x(6)*cos(2*pi*time_lag*0.370)+x(7)*sin(2*pi*time_lag*0.370)-xcov_a_t_s;
    x0=[0;0;0;0;0;0;0];

    [x,ssq,cnt]=LMFnlsq(Eq,x0);

    if ssq/sqrt(sum(xcov_a_t_s.*xcov_a_t_s))>0.01
       warning('xcov_a_t contends to big values, the Levenber-Marquardt fit will fail')
    end

    amp_ss=double([x(2) x(4) x(6); x(3) x(5) x(7)]);
    amp_2(:,:,i)=amp_ss*1e003;
    start_2(:,i)=x(1);
end

clear  x x0 ssq cnt Eq xcov_a_t_s amp_ss

for i=1:1:max(size(amp(1,1,:)))

x=start_2(1,i)+amp_2(1,1,i)*cos(2*pi*time_lag*0.460)+amp_2(2,1,i)*sin(2*pi*time_lag*0.460)+amp_2(1,2,i)
*cos(2*pi*time_lag*0.400)+amp_2(2,2,i)*sin(2*pi*time_lag*0.400)+amp_2(1,3,i)*cos(2*pi*time_lag*0.370)+
amp_2(2,3,i)*sin(2*pi*time_lag*0.370);
    xcov_a_t_qbo_2(:,i)=x;
end

clear x

for i=1:1:max(size(xcov_a_t(1,:)))

    xcov_a_t_s=xcov_a_t(:,i)*1e-003;
```

```
    Eq=@(x) x(1)+x(2)*cos(2*pi*time_lag*0.460)+x(3)*sin(2*pi*time_lag*0.460)-xcov_a_t_s;
    x0=[0;0;0];

    [x,ssq,cnt]=LMFnlsq(Eq,x0);

    if ssq/sqrt(sum(xcov_a_t_s.*xcov_a_t_s))>0.01
       warning('xcov_a_t contends to big values, the Levenber-Marquardt fit will fail')
    end

    amp_ss=double([x(2); x(3)]);
    amp_3(:,:,i)=amp_ss*1e003;
    start_3(:,i)=x(1);
end

clear  x x0 ssq cnt Eq xcov_a_t_s amp_ss

% plotting the single reconstructed peak with smaller stepwiths to
% eliminate the because of  the plotting properties upcoming beatingfrequency
for i=1:1:max(size(amp(1,1,:)))
    x=start_3(1,i)+amp_3(1,1,i)*cos(2*pi*time_lag_2*0.460)+amp_3(2,1,i)*sin(2*pi*time_lag_2*0.460);
    xcov_a_t_qbo_3(:,i)=x;
end

clear x
```

8.5. test_fft_cosx_cosy_2

Testing different couplings of two cosine waves

```
% In this file are thwo cosine waves one with long periode and one with low
% periode coupled by addition and multiplication and are crosscovarianced
% in every possible combination with another coupling
```

```
% After that the 12 different crossvorainainces are fast fourier
% transformed and plotted
% This is done to find out which coupling gives a same sheme of the fft as
% the one which was achieved with crosscovariancing the PW amplitudes with
% the qbo datasets
```

creating the cosine-waves and the x- axes for the plots

```
Z=1:1:576;

X=10*cos(2*pi*Z/12);
Y=10*cos(2*pi*Z/144);

T=1:1:1151;

time_lag=-575:1:575;
```

Executing the xcov and checking the significances via fastlomb

```
xcov_1=xcov(X.*Y,X.*Y,'coeff');
xcov_2=xcov(X.*Y,X+Y,'coeff');
xcov_3=xcov(X+Y,X.*Y,'coeff');
xcov_4=xcov(X+Y,X+Y,'coeff');
xcov_5=xcov(X.*Y,X,'coeff');
xcov_6=xcov(X.*Y,Y,'coeff');
xcov_7=xcov(X+Y,X,'coeff');
xcov_8=xcov(X+Y,Y,'coeff');
xcov_9=xcov(X,X.*Y,'coeff');
xcov_10=xcov(Y,X.*Y,'coeff');
xcov_11=xcov(X,X+Y,'coeff');
xcov_12=xcov(Y,X+Y,'coeff');
```

```
[P_1,f_1,a_1]=fastlomb(xcov_1,T,0,1,4,0.9999);
[P_2,f_2,a_2]=fastlomb(xcov_2,T,0,1,4,0.9999);
[P_3,f_3,a_3]=fastlomb(xcov_3,T,0,1,4,0.9999);
[P_4,f_4,a_4]=fastlomb(xcov_4,T,0,1,4,0.9999);
[P_5,f_5,a_5]=fastlomb(xcov_5,T,0,1,4,0.9999);
[P_6,f_6,a_6]=fastlomb(xcov_6,T,0,1,4,0.9999);
[P_7,f_7,a_7]=fastlomb(xcov_7,T,0,1,4,0.9999);
[P_8,f_8,a_8]=fastlomb(xcov_8,T,0,1,4,0.9999);
[P_9,f_9,a_9]=fastlomb(xcov_9,T,0,1,4,0.9999);
[P_10,f_10,a_10]=fastlomb(xcov_10,T,0,1,4,0.9999);
[P_11,f_11,a_11]=fastlomb(xcov_11,T,0,1,4,0.9999);
[P_12,f_12,a_12]=fastlomb(xcov_12,T,0,1,4,0.9999);
```

Plotting the xcovs together with the original couplings

```
figure(1)
subplot(1,2,1), plot(time_lag,xcov_1), title('crosscovariance X.*Y with X.*Y ')
subplot(1,2,2), loglog(f_1,P_1), title('fastlomb crosscovariance X.*Y with X.*Y ')

figure(2)
subplot(1,2,1), plot(time_lag,xcov_2), title('crosscovariance X.*Y with X+Y ')
subplot(1,2,2), loglog(f_2,P_2), title('fastlomb crosscovariance X.*Y with X+Y ')

figure(3)
subplot(1,2,1), plot(time_lag,xcov_3), title('crosscovariance X+Y with X.*Y ')
subplot(1,2,2), loglog(f_3,P_3), title('fastlomb crosscovariance X+Y with X.*Y ')

figure(4)
subplot(1,2,1), plot(time_lag,xcov_4), title('crosscovariance X+Y with X+Y ')
subplot(1,2,2), loglog(f_4,P_4), title('fastlomb crosscovariance X+Y with X+Y ')
```

```
figure(5)
subplot(1,2,1), plot(time_lag,xcov_5), title('crosscovariance X.*Y with X ')
subplot(1,2,2), loglog(f_5,P_5), title('fastlomb crosscovariance X.*Y with X ')

figure(6)
subplot(1,2,1), plot(time_lag,xcov_6), title('crosscovariance X.*Y with Y ')
subplot(1,2,2), loglog(f_6,P_6), title('fastlomb crosscovariance X.*Y with Y ')

figure(7)
subplot(1,2,1), plot(time_lag,xcov_7), title('crosscovariance X+Y with X ')
subplot(1,2,2), loglog(f_7,P_7), title('fastlomb crosscovariance X+Y with X ')

figure(8)
subplot(1,2,1), plot(time_lag,xcov_8), title('crosscovariance X+Y with Y ')
subplot(1,2,2), loglog(f_8,P_8), title('fastlomb crosscovariance X+Y with Y ')

figure(9)
subplot(1,2,1), plot(time_lag,xcov_9), title('crosscovariance X with X.*Y ')
subplot(1,2,2), loglog(f_9,P_9), title('fastlomb crosscovariance X with X.*Y ')

figure(10)
subplot(1,2,1), plot(time_lag,xcov_10), title('crosscovariance Y with X.*Y ')
subplot(1,2,2), loglog(f_10,P_10), title('fastlomb crosscovariance Y with X.*Y ')

figure(11)
subplot(1,2,1), plot(time_lag,xcov_11), title('crosscovariance X with X+Y ')
subplot(1,2,2), loglog(f_11,P_11), title('fastlomb crosscovariance X with X+Y ')

figure(12)
subplot(1,2,1), plot(time_lag,xcov_12), title('crosscovariance Y with X+Y ')
subplot(1,2,2), loglog(f_12,P_12), title('fastlomb crosscovariance Y with X+Y ')
```